Computer Simulation of the Relationship between Flat Service and Service Return in Tennis Singles

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Abstract

In tennis singles matches, the serving players try hit an ace by placing the ball to their advantage. In contrast, the receiving players try to position themselves for the best defence to return the ball in time and avoid losing points. This study is an attempt to establish a mathematical model on how to attack by optimally placing a flat service and to return the service with the optimal defence position. Four members of a college tennis team (height: 175.5±2.1 cm; weight: 70.3±5.4 kg; age: 22±3.1 years) were the subjects of this study. The experiment was conducted in three stages. First, a flight algorithm based on the physics of the flat service was established. Next, spline functions were used to construct an algorithm for the 3D space available for defence during service return. Finally, data on the time of service return were collected through a high-speed video camera and then keyed into a computer program to test and simulate the flight and defence algorithms. When the serving player increased the impact speed, the receiving player was unable to hit it back. Also, when the placement was close to the receiving player’s lateral sides, the serving player also increased his chances of winning a point. However, if the service bounced on the ground right in front of the receiving player, it was more likely to be returned. The placement findings were defined as the U effect in this study. This experiment also defined the receiving player’s original defence space and helped identify positions vulnerable to attack, which enabled successful service returns. In conclusion, the simulation results of this experiment correspond to real conditions; therefore, the model established in this study is qualified to be a training tool for tennis players in serving and service returning.

Keywords: Algorithm, U effect, Defensive space

Introduction

Tennis players who excel at serving have an edge over others, while those who do not, have difficulty in becoming elite [1]. Although different body heights, arm lengths and muscular strength lead to different serving postures, players are supposed to follow the basic principles of a high contact point, readiness to swing the racket, and an appropriate shift of the centre of gravity. In contrast, the receiving player should focus on preventing the serving player from obtaining any advantages [1].

Steele et al. conducted a wind tunnel experiment on tennis ball surfaces [2]. Four kinds of balls were tested in that experiment: unused new balls and those used over three different time periods. They found that the balls had different drag coefficients. The drag coefficient for the unused new ball was 0.7–0.9. Wang found different coefficients of restitution (CR) for different tennis courts. A red clay court had a CR of 0.69 [3]. Wong found that the speed of the tennis ball when it touched the ground did not tremendously alter the CR [4]. Haake et al. conducted an experiment on the impact of the ball on the racket and found that CR was reduced with increasing impact speed [5]. This relationship may stem from the vibration damping produced by the racket handle and strings.

Several studies investigated the velocity of the tennis ball. Akutagawa and Kojima found that the rotation moment produced by the trunk and hip joints had a direct influence on the speed of single-handed and two-handed backhand tennis strokes [6]. Tanabe and Ito used 3D photography to analyze the velocity of a tennis service [7]. They found that the impact speed of the tennis service was determined by the angle and angular velocity of the shoulder joint. In his research on tennis serves, Tasi took four Taiwanese college tennis players as participants [1]. He found that the maximum serving speeds ranged from 50 to 65 m/s.

For computer simulations, Chiu used a numerical algorithm to simulate the flight distance for badminton [8], tennis, volleyball, and soccer. He found that using a 0.0001s time interval, the error in the results yielded by the algorithm was negligible.

When holding service, a tennis player can serve twice at most. For the first service, the flat service is often used because it is faster and allows the receiving player less time to react, which reduces the return rate. However, the focus of the second service is on the success percentage and placement. Services with topspin, placed on the receiving player’s lateral
sides, are adopted to prevent successful return strokes [1]. In other words, accurate prediction of the impact speed and ball placement is crucial to the serving player. However, few previous studies have explored the relationship between the offensive and defensive strategies for service and service return, respectively. This study is an attempt to establish a mathematical model for service and service return to increase the tactics and strategies available to tennis players. In the future, the proposed algorithm can be developed into computer software, which can be used by tennis coaches and players to acquire practical strategies.

**Methods**

**Subjects**

The subjects were four male members of the tennis team at National Taiwan Chung Hsing University (height: 175.5±2.1 cm; weight: 70.3±5.4 kg; age: 22±3.1 yr). In standing position, their maximal height for the ball contact point was 275.0±2.4 cm. They were all right-handed, and they had participated in matches for more than three years. They had no history of injury for the last six months, and they showed no sign of soreness or pain during the experiment.

**Mathematical Modelling**

The centre of the tennis court was taken to be the Cartesian coordinate system $OXYZ$ (Figure 1a). The serving player was on the left side of the court, while the receiving player was on the right side. $t_k (k = 0,1,\ldots, n)$ represented a certain time point during consecutive flight times. The service was assumed to be released at time point $t_0$. If the receiving player could return the serving before time point $t_k$, which is when the ball touches the ground a second time, this meant that the service was returned successfully. If not, the serving player gained one point.

![Figure 1](image)

**Algorithm for the flight of the tennis ball**

To better understand the keys to a successful service and service return, two algorithms were constructed for simulations. One of the algorithms dealt with the flight of the tennis ball. A Cartesian coordinate system $OXYZ$ was set in the centre of the court (Figure 1c). Next, a translation coordinate system $OX'Y'Z'$ was set up, and its coordinate origin was defined as the tennis ball’s centre of mass (COM) $p_0$; this was also the initial position vector. $\alpha$ referred to the azimuth angle of the service, $\beta$ was the angle of elevation, and $V_0$ was the initial velocity vector relative to air (Figure 1c).

The flight process began with the serve and ends when the ball touches the ground a second time. The initial position vector of the tennis was assumed to be $p_0 = [X_o, Y_o, Z_o]$. The position vector of COM at $t_k$ was assumed to be $p_k = [X_k, Y_k, Z_k]$. The horizontal flight distance from $P_0$ to $P_k$ was $d = \sqrt{(X_k - X_o)^2 + (Y_k - Y_o)^2}$. In $P_0$, the elements $X_o, Y_o$ represented the position points of plane $XY$ in the Cartesian coordinate system, and $Z_o$ was the height of the contact point.

For the flight process, two neighbouring time points $t_k$ and $t_{k+1}$ were randomly selected. The interval between them was assumed to be $\Delta t = t_{k+1} - t_k$. When $t_k$ and $t_{k+1}$ approximate each other, the movement of COM from $t_k$ to
The position vector $P_{k+1}$ at $t_{k+1}$ was computed with equation (1):

$$P_{k+1} = P_k + V_k \Delta t + \frac{1}{2} A_k \Delta t^2$$

In equation (1), $V_k = [V_{kx}, V_{ky}, V_{kz}]^T$ and $A_k = [A_{kx}, A_{ky}, A_{kz}]^T$ referred to the velocity and acceleration vector of COM, respectively. $V_{k+1}$ and $A_k$ could be written as follows:

$$V_{k+1} = V_k + A_k \Delta t$$

$$A_k = F_k / m$$

In equation (3), $F_k$ represented two external forces, gravity and drag force, that were imposed on the tennis ball during its flight; $m$ referred to the mass. The initial velocity vector was represented by $V_0$, and $V_0 = A_{z} V_s (\alpha, \beta) [V_{s0}, 0, 0]^T$ (Figure 1c). $V_s$ represented the release speed, and $A_{z} \ (\alpha, \beta)$ represented the rotation coordinate transformation matrices:

$$A_{z} (\alpha, \beta) = $$

The flight of the tennis ball has three forces acting on it: gravity, drag, and lift (Figure 2). Since a flat service has no spin, the lift force was 0 in this study. Meanwhile, $F_k$ in $t_k$ could be written as follows:

$$F_k = \frac{1}{2} \rho C_d A_k \|V_k\| u_k + m g$$

In equation (5), the density of air $\rho$ was 1.23 kg/m$^3$ at standard atmosphere pressure [12]. $A_k$ referred to the drag area of the tennis, the drag coefficient $C_d$ was 0.78 [2], the gravity acceleration vector $g = [0, 0, -9.81]$ m/s$^2$, $u_k$ was the unit vector for the drag in $t_k$, and $u_k = V_k / \|V_k\|$. $Z_k$ represented the coordinate value along the Z-axis, and $r$ referred to the radius of the ball; therefore, it was assumed that $Z_k - r \leq 0$. Assuming that the ball touched the ground at $t_k$, the velocity vector could be written as $V_k = [V_{kx}, V_{ky}, -\epsilon V_{kz}]^T$. The direction of the velocity components for $V_k$ on the Z-axis was opposite to the original direction, so the velocity of $V_k$ was equivalent to the original velocity components multiplied by the coefficient of restitution $\epsilon$.

Figure 2. Model of defensive space. (a) The heights of the four planes A, B, C and D were $h_0$, $h_2$, $h_1$ and $h_4$ respectively. (b) Time parameters for the defensive space.

Requirements for successful flight over the net

According to International Tennis Federation rules [13], that ball radius $r$ should be 0.033 m, the net height $H_{net}$ is 0.914 m, and the distance between the side and centre lines is 4.115 m. Consequently, the requirements for successful flight over the net in time period $t_k$ are: $|X_k| \leq 0.05m$, $|Y_k| \leq 4.115m$, and $Z_k - r \geq H_{net}$ ($|X_k| \leq 0.05m$: range of error tolerance).

Requirements for valid placements

The position vector of the target placement was assumed to be $Q = [X, Y, 0]^T$. The azimuth of the service could then be computed from the two position vectors $P_i$ and $Q$:

$$a = \tan^{-1}\left(\frac{Y - Y_0}{X - X_0}\right)$$

Figure 2.
The angle of elevation was $\beta$, and $\beta = \beta_{\text{min}} + m \times \Delta \theta$ ($m = 0, 1, \ldots, n$; $\Delta \theta = (\beta_{\text{max}} - \beta_{\text{min}})/n$; $n$ is an integral; $\Delta \theta$: angle of step). The minimal angle of elevation $\beta_{\text{min}}$ was input into the equation, and the angle was gradually increased from $\beta_{\text{min}}$ to $\beta_{\text{max}}$ to calculate the horizontal flight distance and locate $\beta$ for the target area. For $t_k$, which was when the ground is first touched, $ds$, the error for horizontal distance between $P_k$ and $Q$, could be computed using equation (7):

$$ds = \left[ (X_k - X)^2 + (Y_k - Y)^2 \right]^{1/2}$$

(7)

An error tolerance of 0.05 m was presumed for $ds$ (Figure 1a). Therefore, the conditions for when the ball first touches the ground are $Z_t - r \leq 0$ and $|ds| \leq 0.05$ m.

**Figure 3.** (a) Experiment setting. (b) Direction light plate. (c) Prepared position of the receiving player.

**Defence algorithm**

The defensive space referred to in this study is the maximal space within which the service can be returned before it touches the ground a second time. The centre of the tennis court was the origin of the Cartesian coordinate system $OXYZ$ (Figure 1). The serving player was on the left side of the court; he hit the ball from the origin of the translated coordinate system $OS_1YS_1ZS_1$. The receiving player was on the right side of the court; he prepared at the origin of the cylindrical coordinate system $OXSYSZS$ to return the service (Figures 1 and 3). The calculation steps for the entire process are as follows:

**Step 1:** The flight equation was adopted to calculate the position vector $P_k$ relative to the Cartesian coordinate system $OXYZ$ at $t_k$ (Figures 1a and 1c). $P_k$ is then coordinately transformed into $R = [r_k, \theta_k, h_k]$, the position vector of the cylindrical coordinate system $OXSYSZS$ ($r_k$: radius; $\theta_k$: azimuth; $h_k$: height).

**Step 2:** The defensive space for the service return was established (Figures 2a and 2b). The prepared position of the receiving player was assumed to be the origin of the cylindrical coordinate system $OXSYSZS$. The defensive space consisted of four planes ($A$, $B$, $C$ and $D$) which paralleled the $OXS$ plane. The heights of each plane in the cylindrical coordinate system $OXSYSZS$ were indicated by $h_1$, $h_2$, $h_3$ and $h_4$. Each plane had 25 contact points (Figures 2 and 3a), so together the four planes had 100 contact points.

**Step 3:** Plane $A$ was used as an example here (Figure 2a). There were 25 contact points on plane $A$ (Figure 2). One of the contact points was the origin. The other 24 contact points were distributed evenly in eight direction axes: east ($E$), west ($W$), south ($S$), north ($N$), northwest ($NW$), southwest ($SW$), southeast ($SE$) and northeast ($NE$). Each direction axis was comprised of three contact points and the origin $r_0$. The 24
contact points formed the three concentric circles $S_1$, $S_2$, $S_3$, whose radii were $r_1$, $r_2$, and $r_3$, respectively.

**Step 4:** The movement of the receiving player along the $Xs$ axis of the plane $A$ was elaborated in Figure 2. The time the receiving player spent moving from the origin of the cylindrical coordinate system $OXsYsZs$ to the plane $S_0$ was assumed to be $t^{d(0)}_{i,1} [t^{d(0)}_{i,1}(1), t^{d(0)}_{i,1}(2), t^{d(0)}_{i,1}(3), t^{d(0)}_{i,1}(4), t^{d(0)}_{i,1}(5), t^{d(0)}_{i,1}(6), t^{d(0)}_{i,1}(7), t^{d(0)}_{i,1}(8)]$. Spline interpolation was applied to the collected data at random $t^{d(0)}_{i,1}$ to construct a set of spline functions. This way, the time spent by the receiving player in moving from the origin to $S_0$ along the $Xs$ axis of plane $A$ could be calculated with equation (8):

$$t^{d(0)}_{i,1} = \text{Spline}(R_i, t^{d(0)}_{i,1}, r_i)$$

**Step 5:** Following step 4, the time the receiving player spent in moving from the origin of cylindrical coordinate system $OXsYsZs$ (Figure 2b) to $R_{00}$ any of the eight direction axes was assumed to be $t^{d(0)}_{i,2} [t^{d(0)}_{i,2}(1), t^{d(0)}_{i,2}(2), t^{d(0)}_{i,2}(3), t^{d(0)}_{i,2}(4), t^{d(0)}_{i,2}(5), t^{d(0)}_{i,2}(6), t^{d(0)}_{i,2}(7), t^{d(0)}_{i,2}(8)]$. The azimuth angle for any of the eight directions is $\theta = [0, \pi/4, 2\pi/4, 3\pi/4, 5\pi/4, 7\pi/4, 2\pi] / 4$ (Figure 2). Spline interpolation is then applied to the above two sets of collected data to calculate the time the receiving player spent moving at the azimuth angle $\theta_k$ from the origin of $OXsYsZs$ to $R_{k0}$.

$$t^{d(0)}_{i,2, \theta_k} = \text{Spline}(\theta, t^{d(0)}_{i,2, \theta_k})$$

**Step 6:** The time the receiving player spent in jumping to planes $A$, $B$, $C$ or $D$ and moving from the origin of the cylindrical coordinate system $OXsYsZs$ at the azimuth angle $\theta_k$ to $R_{00}$ was assumed to be $t^{d(0)}_{i,3} [t^{d(0)}_{i,3}(1), t^{d(0)}_{i,3}(2), t^{d(0)}_{i,3}(3), t^{d(0)}_{i,3}(4), t^{d(0)}_{i,3}(5), t^{d(0)}_{i,3}(6), t^{d(0)}_{i,3}(7), t^{d(0)}_{i,3}(8)]$. The height of the four planes was represented by $H = [H_1, H_2, H_3, H_4]$, and the time spent in moving from the origin at the angle $\theta_k$ to the plane $S_0 = [x_0, y_0, z_0]$ represented as $t_{00} = [t_{00,1}, t_{00,2}, t_{00,3}, t_{00,4}]$. Spline interpolation was applied to the above two sets of collected data to calculate the time the receiving player spent in moving from the origin to $R(\theta_k, x_0, y_0)$ (Figure 2b):

$$t_{r0b} = \text{Spline}(H, t_{r0b}, r_b)$$

**Step 7:** As shown in Step 6, the receiving player moved from the origin to $R(\theta_k, x_0, y_0)$ to return service; the time spent is represented with $t_{r0b}$. $t_{r0b}$ could be calculated using steps 1 to 6. If $t_{r0b}$ was longer than $t_{00}$, or when the ball touched the ground for the second time, this meant that the receiving player did not intercept the ball and the serving player gained one point.

**Possibility of aching**

The court on the receiving player’s side was divided into 100 rectangles of the same size. The centre of each rectangle was defined as the target placement for the serve. One hundred placements were simulated to determine the possibility of aching.

**Data Collection Procedure**

The collected data came from taking pictures of two subjects returning service with two high-speed video cameras (200 Hz). The origin of the coordinate system $OXsYsZs$ served as the centre of circle $r_0$ ($r_0 = 0$ m) for the three concentric circles $S_1$, $S_2$ and $S_3$, whose radii were $r_1$, $r_2$ and $r_3$ ($r_1 = 3$ m, $r_2 = 8$ m, $r_3 = 18$ m). A direction light plate was placed 25 m north of $r_0$. Two video cameras were placed on either side of $r_0$ 25 m away. The cameras could shoot the subjects’ movement and the signals of the direction light plate. The subject stood on the origin in a prepared position (Figures 3a and 3b) with his heels separated by 0.5 m. The heels were 0.5 m away from the $Ys$-axis, and an isosceles triangle was formed by the origin of $OXsYsZs$ and the two heels (Figure 3b). On the direction light plate, each of the eight directions had an indication light (Figure 3c). The experimenter randomly turned on one of the direction lights and recorded the time for the service return.

For circle $S_0$, each of the eight direction axes had a plumb line (Figure 3) to which four tennis balls ($a$, $b$, $c$ and $d$) were attached at different heights ($h_1 = 0.15$ m, $h_2 = 0.90$ m, $h_3 = 1.70$ m, $h_4 = 2.75$ m). One of the eight direction lights was then randomly pressed (each direction light could only be pressed once). Seeing the light, the subject had to move from the prepared position to the direction the light indicated to hit ball $a$. Ball $a$ was hit in one of the eight directions, and each movement time was recorded. Subsequently, the movement time in hitting balls $b$, $c$ and $d$ in each direction was recorded. The above procedure was applied to circles $S_2$ and $S_3$ to record the movement time spent in hitting balls $a$-$d$ in the eight directions. The time consumed in hitting the four balls $a$-$d$ on $S_0$ was also recorded. In total, 100 balls were hit. Each ball was hit twice, and hits which took less time were chosen as the test value. The subjects hit the balls at intervals of 90–120 seconds.

**Data Analysis**

The computer programming language Borland C++ was used to simulate the flight of the tennis ball and to test the flight and defence algorithms. Four physics parameters had to be confirmed before simulation and testing. One was the drag coefficient. According to the graph developed for the tennis ball’s drag coefficient by Steele et al. [2], the coefficient $C_d$ was 0.78. Another parameter was the coefficient of restitution. According to Wong [4], on a red clay court, the coefficient of restitution for a flat service with no spin was 0.69. The next parameter was the time step for simulating the flight of the tennis ball. The time step was assumed to be $\Delta t \leq 0.0001$ s in this study because the horizontal distance of the fall remained the same (Table 1). The relative error was below 0.002% ($\Delta t \leq 0.0001$ s). The final parameter was angle step $\Delta \theta$ of $\beta$ for locating the placement of the tennis ball. Test results showed that $\Delta \theta = 0.005$° (Table 2) provided the best results since the produced error for adopting $\Delta \theta = 0.005$° to predict the placement was 21.68% ($1 \leq ds \leq 0.05$ m), which was within the acceptable range $ds$. 
Table 1. The numerical time step \( \Delta t \) for the flight algorithm of the tennis ball

<table>
<thead>
<tr>
<th>( \Delta t ) (s)</th>
<th>( T_f ) (s)</th>
<th>Distance (m)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.84000</td>
<td>25.8211</td>
<td>0.091</td>
</tr>
<tr>
<td>0.01</td>
<td>0.84000</td>
<td>25.8933</td>
<td>0.371</td>
</tr>
<tr>
<td>0.001</td>
<td>0.83299</td>
<td>25.8029</td>
<td>0.021</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.83253</td>
<td>25.7982</td>
<td>0.002</td>
</tr>
<tr>
<td>0.00001</td>
<td>0.83320</td>
<td>25.7976</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: \( T_f \) represented the flight time; \( V_s = 45 \text{ m/s}, \alpha = 0, \beta = 0 \), \( Z_0 = 2.75 \text{ m} \). Relative error = 100% \( \times \) (horizontal distance - 25.7976)/25.7976.

Table 2. The error for adopting \( \Delta \theta \) to predict the placement

| \( \Delta \theta \) (deg) | \( \beta \) (deg) | |ids| (m) | Relative error (%) |
|--------------------------|------------------|---------------------|---------------------|---------------------|
| 0.2                      | -4.60            |                     | 0.05868             | 117.36              |
| 0.1                      | -4.60            |                     | 0.03559             | 71.18               |
| 0.05                     | -4.58            |                     | 0.02759             | 55.18               |
| 0.01                     | -4.57            |                     | 0.01623             | 32.46               |
| 0.005                    | -4.57            |                     | 0.01084             | 21.68               |

Note: \( \Delta t = 0.0001 \text{ s}; \ V_s = 45 \text{ m/s}; \alpha = 0; \beta = 0; \) contact position (0m, 0m, 2.75m); placement (12m, 0m, 0m). Relative error = 100% \( \times \) \(|d|/0.05\).

Results and Discussion

Defensive space

The defensive space refers to the maximal range where the receiving player can return service before the ball touches the ground a second time. The success of the service return is determined by the receiving player’s reaction and movement times [1]. The measurement of the defensive space depends on the area of the receiving player’s physical activity [14]. Figure 4 showed that the volume of each subject’s defensive space became larger as the defence time increased (Table 3). In addition, the four subjects had defensive spaces with different styles. Apparent differences could be found in the four spaces at a defence time of 0.6 s. When the defence time increased to 1.8 s, the defensive space was shaped like a pie chart. Subject D had the largest defensive space; his volume reached 445.4 \( \text{m}^3 \) when the defence time was 1.8 s. This result shows that subject D was faster than the other subjects in returning a service. Being speedy in movement meant that the receiving player had more time, which enhanced the possibility of a successful service return.

Table 3. Defensive space of four subjects

<table>
<thead>
<tr>
<th>Subjects</th>
<th>Ts=0.6s</th>
<th>Ts=0.9s</th>
<th>Ts=1.2s</th>
<th>Ts=1.8s</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Vol (m³)</td>
<td>11.4 (A1)</td>
<td>62.8 (A2)</td>
<td>150.0 (A3)</td>
</tr>
<tr>
<td>B</td>
<td>Vol (m³)</td>
<td>16.9 (B1)</td>
<td>75.9 (B2)</td>
<td>168.3 (B3)</td>
</tr>
<tr>
<td>C</td>
<td>Vol (m³)</td>
<td>15.3 (C1)</td>
<td>69.5 (C2)</td>
<td>156.5 (C3)</td>
</tr>
<tr>
<td>D</td>
<td>Vol (m³)</td>
<td>13.8 (D1)</td>
<td>74.7 (D2)</td>
<td>180.7 (D3)</td>
</tr>
</tbody>
</table>

Note: Ts represented the movement time; Vol represented the volume of defensive space (See figure 4).

Table 4. Time subject B spent to receive the stroke

<table>
<thead>
<tr>
<th>Case</th>
<th>Successful receiving</th>
<th>Vs (ms)</th>
<th>Contact point</th>
<th>Placement</th>
<th>Time(s)</th>
<th>Tf(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>X(m)</td>
<td>Y(m)</td>
<td>Z(m)</td>
<td>X(m)</td>
</tr>
<tr>
<td>a</td>
<td>Yes</td>
<td>25</td>
<td>0</td>
<td>-8.0</td>
<td>2.0</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>Yes</td>
<td>25</td>
<td>0</td>
<td>-8.0</td>
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<tr>
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<td>No</td>
<td>25</td>
<td>0</td>
<td>-8.0</td>
<td>2.0</td>
<td>3</td>
</tr>
<tr>
<td>d</td>
<td>Yes</td>
<td>20</td>
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<td>1.5</td>
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<td>e</td>
<td>Yes</td>
<td>30</td>
<td>0</td>
<td>-10.0</td>
<td>2.5</td>
<td>1.5</td>
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<tr>
<td>f</td>
<td>No</td>
<td>40</td>
<td>0</td>
<td>-10.0</td>
<td>2.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Note: Defence position: (0m, 8.5m, 0m). Tf: surplus or deficient time between receiving and the ball touching the ground a second time (See figure 5).
Figure 4. Subjects are represented by A, B, C and D. The serial numbers 1, 2, 3 and 4 represent the defensive spaces for defence times of 0.6, 0.9, 1.2 and 1.8 s, respectively.

**Preliminary test of flight and defence algorithms**

The receiving strokes of the four subjects were simulated. Subject B was used here as an example (Figure 5; Table 4). The player stood in position (0m, -8.0m, 2.0m), released three balls at a speed of 25 m/s, and placed the balls at (0m, 6m, 0m), (1.5m, 6m, 0m) and (3m, 6m, 0m) (Table 4). The algorithm was used to simulate the receiving stroke of subject B and to judge if he was able to intercept the ball (Figure 5; Table 4).

The first ball was placed right in front of subject B (Figure 5a), so subject B was able to successfully return it with the defence time of 0.788 s. Subject B was able to intercept the ball 0.353 s ahead of touching the ground a second time (Table 4). The second ball was placed a little to the left side of subject B. This time, subject B hit the ball 0.140 s ahead of it touching the ground a second time (Figure 5b; Table 4). The third ball was placed far to the left side of subject B. This time, subject B failed to intercept the ball. The defence time was 0.227 s behind the ball touching the ground a second time (Figure 5c; Table 4).

Subject B was then simulated to hit three other balls that were released from the position (0m, -10.0m, 2.5m) at three different speeds—20, 30 and 40 m/s—and had the same placement (1.5m, 6m, 0m). When the release speed was 20 or 30 m/s, subject B could hit the ball (Figure 5d and 5e; Table 4). When the release speed was 40 m/s, subject B was unable to intercept it, and the defence time was 0.134 s behind the ball touching the ground a second time (Figure 5f; Table 4).

The simulation results showed that the closer the placement was to the receiving player’s initial prepared position, the more easily he could receive the stroke. On the other hand, the more distant the ball was placed from the receiving player, the more
difficult it was to hit the ball. The lower the release speed of the stroke, the easier it was to hit the ball. This confirmed the reason why most tennis players used the flat service as the first serve [1], as it is faster and allows the receiving player less time to react and move. Since the findings of the preliminary test on the flight and defence algorithms corresponded to the results of other studies, the mathematical model in this study can be adopted to simulate service and service return.

Figure 5. The release speed Vs of the three balls was 25 m/s (Table 4), and their placements were (0 m, 6 m, 0m), (1.5 m, 6m, 0m), and (3m, 6m, 0m) (Table 4). Subject B was simulated to hit the three balls and obtained the placements shown in (a), (b) and (c). Next, subject B was simulated to hit three other balls released at three different speeds—20, 30 and 40 m/s—with the same placement (1.5m, 6m, 0m). The simulation results are shown in (d), (e) and (f).

Figure 6. Illusion of how subject B returned the service (Vs = 55m/s); contact point of service (CP), position of service contact (-0.10m,-12m, 2.75m).

Table 5. Contact points of service and receiving player B’s initial prepared position

<table>
<thead>
<tr>
<th>Case</th>
<th>Vs (m/s)</th>
<th>X(m)</th>
<th>Y(m)</th>
<th>Z(m)</th>
<th>X(m)</th>
<th>Y(m)</th>
<th>Z(m)</th>
<th>Ace(%)</th>
</tr>
</thead>
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<tr>
<td>a</td>
<td>55</td>
<td>-0.1</td>
<td>-12.0</td>
<td>2.75</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>19</td>
</tr>
<tr>
<td>b</td>
<td>55</td>
<td>-0.1</td>
<td>-12.0</td>
<td>2.75</td>
<td>3</td>
<td>13</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>c</td>
<td>55</td>
<td>-0.1</td>
<td>-12.0</td>
<td>2.75</td>
<td>5</td>
<td>13</td>
<td>0</td>
<td>12</td>
</tr>
</tbody>
</table>

Note: Simulation results of subject B’s service return are presented here(See figure 6).

**Analysis of ace**

Tasi found that the speed of the first service of four tennis champions in category A was between 50 and 65m/s [1] . Therefore, when conducting simulations in this study, a release speed of 55 m/s was adopted; three different positions of service contact were assumed at -0.10, -12 and 2.75 m. The court on the receiving player’s side was assumed to have 100 placements. When receiving player B prepared in the middle position (0m, 13m, 0m) for the service return (Figure 6a), the area he was scored on was found to account for 19% of the whole area (Table 5). When the prepared position of receiving player B was moved to (3m, 13m, 0m), the scoring area was
reduced to 6% (Figure 6b). If the prepared position was moved to the sideline (6m, 13m, 0m), then the undefendable area increased to 12%. The above analysis demonstrated that the serving player would have problems acing when the receiving player prepared in the middle position, while the side placement would lead to acing. The simulation results corresponded to actual tennis match situations [1]; thus, the mathematical model in this study was reaffirmed to be reliable in simulating service and service return.

![Diagram of U effect](image)

**U effect**

The four subjects’ service return (Figure 7) was then simulated further in this study. The release speed was assumed to be 65 m/s. Their initial prepared position was 1 m behind the baseline as the initial position. Their lateral sides extended from (0m, 13m, 0m) to (6m, 13m, 0m), position of service contact (-0.10m, -12m, 2.75m).

Each curve in Figure 7 was dented in the middle, which was termed the ‘U effect’ in this study. The U effect demonstrated that it was easier for the receiving player to return a service placed in front of him and that it was easier for the serving player to ace if ball is placed to the receiving player’s lateral side. The simulation results correspond to actual tennis match situations, proving that the algorithms of this study are reliable and practical. The simulation results also show evidence that the mathematical model in this study can help the receiving player compute his vulnerable areas and the defensive space in accordance with a certain prepared position. The above findings demonstrate that the mathematical model based on the two algorithms can serve as a training tool for tennis players in service and service return.

**Conclusions**

In a tennis match, the receiving player tends to make judgments based on his previous experience. This is neither objective nor effective. This study is an endeavour to provide better strategies by analyzing the release speed, placement and defensive space.

The U effect is found through experiments and simulation with the flight and defence algorithm developed in this study. The U effect demonstrates that it is easier for the receiving player to return a service placed in front of him and that it is easier for the serving player to ace if ball is placed to the receiving player’s lateral side. The simulation results correspond to actual tennis match situations, proving that the algorithms of this study are reliable and practical. The simulation results also show evidence that the mathematical model in this study can help the receiving player compute his vulnerable areas and the defensive space in accordance with a certain prepared position. The above findings demonstrate that the mathematical model based on the two algorithms can serve as a training tool for tennis players in service and service return.

**References**


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