"Time" as a Variable in Demand Equations

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I. The Basic Theory of Time in Economics

Time is moment-in-being, that is "one wave ends and another begins", which revolves around the calendar-axis. It must be thought of as a brief interval of finite element, not a mere point; yet it is also something whose very essence and also whose existence involves its continual movement and continuous evolution. There is for us a moment-in-being, which is the locus of every actual sense-experience, every thought, feeling, decision and action. And as we know that "expectations are a key element in decision-making and thus in the mechanism of evolution of the moment-in-being" (a), so, we must recognize that time is an important element in economics.

An important branch of modern economics is dynamic economics. As Harrod says: "Dynamics will specifically be concerned with the effects of continuing changes and with rates of change in the values that have to be determined" (b), which in other words, it means time is an important element in dynamic economics.

Allen stresses the importance of time in demand by saying "The demand of the market as a rate per unit of time" and "the current price being a function of "t" (c). Even a few decades ago, Marshall recognised that when we analyse demand prices and interpret them, "the first which we have to consider arises from the element of time"(d).

II. Why we Use Time as a Variable that Affects Demand

In general, the majority of data which is used in demand

(a) G.L.S. Shackle, "Time in Economics" Ch. I, 1958
(b) R.F. Harrod, "Towards a Dynamic Economics" P. 8 1948
(c) R.G.D. Allen, "Mathematical Analysis for Economists" p. 434-435, 1956
(d) A. Marshall, "Principles of Economics" P. 109
analysis is of time series nature. But, here, we limit our topic by using time as a direct variable in demand functions, i.e. time series data is not concerned.

Time as a variable that affects demand may arise for the following reasons:

1) Psychological

Under this category we include forces of habit, taste and assumption the part of the consumers that changes may be gradual over time, but considerable seasonal fluctuations are possible.

2) Technological—techniques applicable within consuming unit; Those include factors such as improving of knowledge about possible substitutes, adding service to consumer goods and new nutrition.

3) Institutional

This category includes the social structure and marketing system which may affect demand in the long-run.

The above categories have no empirical data for analytical purposes. So we can only take time as variable into demand function.

This method has been used by many economists when they could not find other relevant empirical data to express time effect in their demand equations, such as Armore. In his study of factors that affect whole-sale prices of fats and oils used in food products he says: "To throw more light on this point, year-to-year changes in the dependent variable were added as a fourth independent variable" so that "the multiple coefficient of determination was raised from 0.92 to 0.96, so that the unexplained variation was reduced from 8 to 4 percent, and a statistically significant partial coefficient of determination of 0.52 was obtained for the new variable." (e)

III. Conditions or Limitations

In demand analysis, time is frequently introduced as a variable into a function or equation. But, first of all, we must consider their conditions or limitations which are as

(e) Armore, Sidney J. "The demand and price structure for food fats and oils" pp. 56-58, 1953
follows;
1) The factor must be one which cannot be measured
2) No data are available
3) The variation of this factor is continuous and systematic over time
4) The time effect is believed to be linear, or curvilinear to a moderate degree only. If the time effect is strongly curvilinear—for example, U-shaped—then a nonsignificant coefficient may be obtained which is, in a sense, misleading.

If these conditions are to be fitted, then a time variable should be included in the initial analysis. At last, we must test it, if (5) its partial regression coefficient fails to differ significantly from zero, it may be omitted.

IV. Methods and Steps

In many cases while formulating the model, the research worker is uncertain about the importance of time effect. Thus time probably should be omitted from the initial analysis. But the unexplained residuals should be plotted against time to determine whether they exhibit a nonrandom pattern. If they do, an attempt should be made to discover an economic cause for the pattern. If no additional variable that is related to the unexplained residuals can be found but an explanation of the time trend in terms of technological or institutional development is available, then a time variable should be introduced into the analysis through the use of an appropriate graphic or mathematical relationship.

The steps used are as follows:-
1) The unexplained residuals for the years involved are plotted against time, and a free-hand is drawn through them that coincides with the assumed effect of the explanatory cause.
2) Deviations from this trend are substituted for the original deviations on each of the partial charts.
3) If a change in the slope or general shape of the relation is suggested, and this appears to be in line with expectations based on economic theory and knowledge
about the item, a graphic adjustment in the relation is made.

4) An appropriate adjustment is made in the various mathematical coefficients obtained from the original analysis.

In analysis, there are two types of equations which can be used; one is arithmetical or non-logarithmic equation, and the other is logarithmic equation or power function. All these are shown as follows:

1) Arithmetical equation

\[ P_i = a + (b + b't)Q_i + (c + c't)D_i + (d + d't)Q_{i-1} + (e + e't)t \]

where \( i = \) time; \( p = \) price; \( q = \) quantity sold and \( D = \) consumer income

If the coefficient on the cross-product terms \( b', c', d', \) and \( e' \) differ from zero by a statistically significant amount, the coefficients with the basic demand equation can be assured to change in a systematic way over time (f).

In general, this arithmetical equation can be written in the following form:

\[ Y = a + b_1 X + b_2 t \]

2) Logarithmic equations (g)

In analysis for which the other variable are converted to logarithms, time \( t \) frequently is used either in a converted or an unconverted form. When it is converted, the first year should be assigned the number \( 1 \), as the logarithm of zero is undefined. The following equations illustrate the effect of these alternative treatment:

(a) Use of converted data:

\[ \log Y = \log a + b \log X + c \log t \]

\[ Y = aX^b t^c \]

Here \( C \) normally is close to zero and may be positive or negative. As \( t \) increases, we raise a progressively larger to a certain power.

Slightly altered in form of equation.

(g) R. J. Foote, Op. Cit. PP. 40-1
(b) Use of unconverted data:

\[ \log Y = \log a + b \log X + (\log c) t \]

\[ Y = aX^bc^t \]

Here \( c \) (the antilog of \( \log c \)) normally is close to 1 and always is positive, but \( \log c \) may be positive or negative and normally is close to zero. As \( t \) increases, we raise \( c \) (a constant) to a progressively larger power.

Results of the two approaches are shown in Fig. 1.

Section A. Power functions obtained when \( t \) is converted to logarithms.

Section B. Exponential functions obtained when \( t \) is not converted.

Fig. 1. Alternative power and exponential functions of time.

In effect, these alternative approaches give (a) a power function in \( t \) and (b) an exponential function in \( t \). The power function increases or decreases more rapidly for small values of \( t \) than does the exponential function, but it soon becomes nearly flat if \( c \) is less than unity. The exponential function never declines below zero, and hence negative time trends become flatter as \( t \) becomes very large; but when it is used for an increasing time trend, it increases more rapidly as \( t \) increases. The typical time trend is one that increases rapidly during a period of growth and then tends to flatten out. Thus the power function, which relates that \( t \) be converted to logarithms, appears more logical for most problems. The wide variety of alternatives permitted by its use is a further advantage.

V. Applications

We use time as a variable in demand function, generally, to measure either seasonal fluctuation or longtime trend. The
seasonal fluctuation here is not only meant by a quarter of year but also including monthly, weekly or daily (e.g. in Saturday, perhaps more goods to be consumed) fluctuations. The following examples illustrate their applications:

1) Seasonal fluctuation

This example is to analyse post war United Kingdom exports to the dollar area. The equation is shown as follows: (h)

\[ x_{st} = \alpha_0 + \alpha_1 \left( \frac{P_s}{P_d} \right)_{t-1} + \alpha_2 P_{st1} + \beta_1 Q_{1t} + \beta_2 Q_{2t} + \beta_3 Q_{3t} + U_t \]

Where \( x_{st} = \) British export to the dollar area

\( \left( \frac{P_s}{P_d} \right) = \) relative prices

\( P_s = \) the level of production in the dollar area

\( Q_{1t} = \) Seasonal factors

\( Q_{1t} = 1 \) in first quarter periods  
\( = 0 \) in all other periods

\( Q_{2t} = 1 \) in second quarter periods  
\( = 0 \) in all other periods

\( Q_{3t} = 1 \) in third quarter periods  
\( = 0 \) in all other periods

\( U_t = \) Disturbance

In each of the first three quarters of any year only one of the \( Q_{1t} \) variables is non-zero. The associated coefficient \( \beta_1 \) shows how the level of the whole equation must be adjusted for that quarter's seasonal influence. In the fourth quarter, all the \( Q_{1t} \) are zero, and the level of the equation is given by \( \alpha_0 \).

The result is

\[ x_{st} = 211.63 + 0.91 \left( \frac{P_s}{P_d} \right)_{t-1} + 2.27 \left( \frac{P_s}{P_d} \right)_{t-1} - 14.86 Q_{1t} + 0.53 Q_{3t} + 1.00 Q_{3t} \]

Thus the normal seasonal pattern, after taking into account the seasonal movements of prices and production, indicates a low point in the first quarter following Christmas sales, and a high point in the third market just preceding Christmas.

From this example, one point must be concerned by us.

In the analysis at seasonal fluctuation, we always use zero-one variable. It means that such a variable has a value of 0 in one period and a value of 1 in another period to indicate the extent to which the dependent variable is larger or smaller in this period.

2) Long-time trend (i)

Allen, in his study of dynamic forms of demand and supply functions, puts the course of prices over time as follows:

\[ P(t) = \bar{P} + (P - \bar{P})e^{\lambda t} \]

where \( \bar{P} = \) equilibrium Price = \( \frac{b-\beta}{\alpha-a} \), \( \lambda = \frac{\alpha-a}{c-Y(j)} \),

\( P_0 = \) the initial price

Here \( t \) is used in an unconverted form. So, the important term in the expression for \( P(t) \) is \( e^{\lambda t} \). This term increases and tends to infinity, or decreases and tends to zero, according as \( \lambda \) is positive or negative. Since we have taken \( (\alpha-a) \) as positive, the sign of \( \lambda \) is governed by that of \( (c-Y) \). Hence

(a) If \( Y>c, \lambda \) is positive and the price steadily diverges from the static equilibrium value \( P \) as time goes on.

(b) If \( Y>c \), \( \lambda \) is negative and the price steadily approaches the static equilibrium value \( P \) as time goes on.

3) Seasonal fluctuations and long-time trend in one equation

In "First Steps to An Analysis of the Rural Credit Market", Jarrect and Dillon said:

(a) A strong time trend is present in all series on advances.

(b) There is a significant seasonal component in the demand for advances from the pastoral houses.

For example. The demand equation of woolgrowers from Pastoral Finance Companies is as follows:

\[ C_w = 26.37 + 0.08Z_{11} + 5.47Z_{18} + 4.105Z_{15} - 4.88Z_{41} \]

where \( C_w = \) credit demand of woolgrowers from PFC

(i) R.G.D. Allen, op. cit. pp. 434-438

(j) Demand \( x = ap(t) + b + cp'(t) \)

Supply \( x = \alpha p(t) + \beta + \gamma p'(t) \)

In the normal case, the constant \( a \) and \( \beta \) are negative and \( b \) and \( \alpha \) are positive.

-265-
$Z_{11} =$ parity ratio for wool growers
$Z_{13} =$ seasonal dummy (Dec = 1)
$Z_{15} =$ time (1 to 27)
$Z_{41} =$ "maximum average interest rate on non-rural overdrafts"
regarded as exogenous

BIBLIOGRAPHY

2. R. F. Harrod, "Towards a Dynamic Economics" 1948
8. K. A. Fox, "The Analysis of Demand for Farm Products"
9. H. Wold, "Demand Analysis" 1962
11. F. G. Jarrett and J. L. Dillon, "First Steps to an analysis of the Rural Credit Market" (given article)