The Conceptual Framework of The Taiwan's Hog Packing Industry

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1 Theoretical Background

1.1 Development of Interregional Trade and Location Theory

The international trade theory, location theory, and neoclassical equilibrium theory have been developed rapidly in the Western society but not in the Orient.

Adam Smith, the first economist, developed the concept of absolute advantage in the international trade theory. Ricardo developed the principle of comparative advantage which states: Each country would gain by exporting that commodity in which comparative costs were lower.

Location theorists such as Von Thuenen, Weber, Hoover, Loesch, Isard, and Dunn concentrate on the space factor rather than just the classical trade theory. And they try to combine the

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5. Edgar M. Hoover, Location Theory And Shoe And Leather Industries, Cambridge, Massachusetts, 1937.
concept of the location theory with the ideas of the theory of firm and general equilibrium analysis.

1.2 The Interregional Activity Analysis Model

The neo-classical economists such as Baumol\(^9\), Beckman\(^10\), Enke\(^11\), Koopmans\(^12\) and Samuelson\(^13\), extended traditional equilibrium analysis by introducing transportation into the problem to cover the space dimension, emphasizing the use of linear programming models to declare the production, trade and price relationships of a multi-location economy.\(^{14}\)

The Enke-Samuelson spatial equilibrium models have found widespread application in empirical analyses. The principal characteristics of this approach are that (1) space is no longer viewed as a continuous plane, but regions are defined as discontinuous points in space in which all economic activity is assumed to take place; and that (2) specifications of the problem are designated in an optimizing framework which explicitly builds the price mechanism into the model and subsequently introduces the simultaneous and interdependent nature of economic activity.\(^{15}\)

Spatial equilibrium analysis, however, does not cover all aspects of the location theory. It is necessary to incorporate production and consumption activities into the programming formulation. Koopmans extended the linear activity analysis model of production and allocation to include trade and transportation activities.\(^{16}\) Beckman and Marschak building on Koopmans contribution developed a general model for a firm operating branch plants.

King and Schrader\(^{17}\) developed an interregional factor-product model for the feeder cattle industry of the United States. Their analysis specified regional demands for beef and set forth an interative procedure to bring price and quantities into equilibrium.\(^{18}\)

Within the context of interregional activity analysis, Takayama and Judge have shown that "in the case where regional demands for final products are represented by price dependent linear functions, the concept of maximizing net consumer surplus can be used to deduce the price conditions of spatial equilibrium and the general interregional activity analysis model can be formulated as a quadratic programming problem.\(^{19}\) Equilibrium prices, quantities, and commodity flows are determined directly in the quadratic programming formulation.

2. The Mathematical Model

2.1 Assumptions

To provide for the practical application of a theoretical model, it is necessary to formulate specific definitions and restrictive assumptions which reflect conditions of an operational economic model. The specific assumptions as they apply to the Taiwan's pork packing industry are explicitly stated:

1. One of the components of Taiwan's industry is the meat packing industry; specifically, the pork packing industry is the subject of analysis.

2. A known non-negative quantity of slaughter hogs is given for the n regions of the multi-regional pork packing industry.

3. It is assumed that the production activities in each region are technologically uniform among regions. The rate at which slaughter hogs are transformed into carcasses is constant for all regions and levels of production.


4. Slaughter hogs are assumed to be homogeneous in type, grade, and quality for all regions.

5. The product, pork carcasses, is assumed in non-negative quantities in each of the n regions. The regional demands are predetermined.

6. Each region has some non-negative capacity for slaughtering hogs and these capacities are known for any time period.

7. A base trading point in each region has been selected at which the supply of slaughter hogs, demand for fork carcasses, and the slaughtering capacities are assumed to be concentrated.

8. Slaughtering costs per head of hog are known in time period t for each region and they differ among regions.

9. All possible trading regions are separated by a transport cost per head of hog and pork-carcass. These transfer costs are known for time period t. The unit transportation costs are independent of the volume of slaughter hog flow and pork carcass flow.

10. No distinction is made between the regions as to methods of slaughtering hogs and consumption of pork-carcass. Thus slaughtering plants and consumers are indifferent about the source of supply. Competitive behavior is stipulated for all participants in the industry.

11. Finally, it is assumed all pork carcasses are traded in competitive market.

2.2 Notations

The following mathematical notation is used:

- $i,j$ denote regions; $i,j = 1,2, \ldots, 9$.
- $t_{ij}$ the unit transport cost for shipment of the hog carcasses (M.T.) between regions $i$ to $j$.
- $q_{ij}$ the quantity of the pork carcasses (M.T.) shipped from region $i$ to $j$ by freezing containers.
- $S_i$ the unit cost of slaughtering hogs (NT$)
- $T_{ij}$ the unit transport cost of the live hogs (NT$) per metric ton.
- $H_i$ the quantity of slaughter hogs produced in region $i$ (Metric Tons).
the live hog shipment from regions \( i \) to \( j \) (M.T.).

- \( C_i \): the slaughtering and processing capacity in region \( i \) (M.T.).

- \( D_i \): the demand for hog carcasses in region \( i \) (M.T.).

- \( A_i \): the level or amount of hog slaughtering (M.T.) in region \( i \).

- \( P_i \): the dressing percentage of the live hog; the nine regions use the same 80 percent as authorized by the government.

- \( g_i \): denote net availability in region \( i \), \( \ell = 1, 2, 3, 4 \).

### 2.3 The Formal Model

Given the restrictive assumptions and using the foregoing notation, the problem may be stated mathematically as follows: To maximize

\[
T = - \sum_{i=1}^{9} \sum_{j=1}^{9} t_{ij} q_{ij} - \sum_{i=1}^{9} s_i A_i - \sum_{i=1}^{9} \sum_{j=1}^{9} T_{ij} Q_{ij}
\]

subject to

\[
g_i = P_i A_i - \sum_j q_{ij} \geq 0
\]

The quantity of pork shipment from the region \( i \) must be less than or equal to the pork equivalent of the number of hogs slaughtered. \( \ell = 1, 2, \ldots, 9 \)

\[
g_i = H_i - \sum_j (Q_{ij} - Q_{ij}) - A_i \geq 0
\]

The level or amount of slaughtering in region \( i \) must be less than or equal to the number of hogs available for slaughter in the region, plus or minus adjustments for live hog shipments. \( \ell = 1, 2, \ldots, 9 \)

\[
g_i = C_i - A_i \geq 0
\]

The level of slaughtering in region \( i \) must be less than or equal to the regional slaughtering capacity.

\[
g_i = P_i A_i + \sum_j q_{ij} - \sum_j q_{ij} - D_i = 0
\]

The quantity of the pork carcasses available in a region,
adjusted by plus or minus shipment of pork carcasses, must equal
the demand for the pork carcasses in region i. (j=1,2,........9).

(6) q_{ij}, A_i, Q_{ij} \geq 0

All selected variables must be non-negative.

3. The Optimality Conditions

Given the above specifications, in order to derive the conditions
necessary for the minimization of transportation and slaughtering
costs subject to the constraints (2) to (6), the following Lagrangean
is formed:

\begin{align*}
(7) \quad \phi(x, u) &= -\sum_i \sum_j t_{ij}q_{ij} - \sum_i S_iA_i - \sum_i \sum_j T_{ij}Q_{ij} \\
&+ \sum_i U_i (C_iA_i - \sum_j Q_{ij}) + \sum_i U_i^P (H_i - \sum_j (Q_{ij} - Q_{ij}^P)) \\
&- A_i) + \sum_j M_j^j(C_j - A_j) + \sum_i U_i^P (P_iA_i + \sum_j q_{ij} \\
&- \sum_j q_{ij}^P - D_i)
\end{align*}

Making use of the Kuhn-Tucker theorem, the necessary
(optimality) conditions which must hold are:

\begin{align*}
(8) \quad \frac{\partial \phi}{\partial q_{ij}} &= -U_i^P - \bar{U}_i^P - t_{ij} \leq 0 \quad \text{and} \quad \left(\frac{\partial \phi}{\partial q_{ij}}\right)_{q_{ij}} = 0 \\
(9) \quad \frac{\partial \phi}{\partial A_i} &= U_i^P + \bar{U}_i^P - \bar{U}_i^P - S_i - M_i^P \leq 0 \quad \text{and} \quad \left(\frac{\partial \phi}{\partial A_i}\right)_{A_i} = 0 \\
(10) \quad \frac{\partial \phi}{\partial Q_{ij}} &= \bar{U}_i^P - \bar{U}_i^P + T_{ij} \leq 0 \quad \text{and} \quad \left(\frac{\partial \phi}{\partial Q_{ij}}\right)_{Q_{ij}} = 0 \\
(11) \quad \frac{\partial \phi}{\partial U_i^P} &= Q_i^P - P_iA_i - \sum_j Q_{ij} \geq 0 \quad \text{and} \quad \left(\frac{\partial \phi}{\partial U_i^P}\right)_{U_i^P} = 0 \\
(12) \quad \frac{\partial \phi}{\partial U_i^Q} &= Q_i^P - A_i \geq 0 \quad \text{and} \quad \left(\frac{\partial \phi}{\partial U_i^Q}\right)_{U_i^Q} = 0 \\
(13) \quad \frac{\partial \phi}{\partial U_i^A} &= Q_i^P - C_i - A_i \geq 0 \quad \text{and} \quad \left(\frac{\partial \phi}{\partial U_i^A}\right)_{U_i^A} = 0
\end{align*}
Following the usual economic interpretation procedure of defining the U's in the Lagrangean (7) as efficiency prices and rents, the conditions (8) through (14) spell out the characteristics of the internal price and rent system that are consistent with an efficient production and allocation system. In order to derive an efficient production and allocation program, these conditions state that regional values and rents must be such that:

1. Profits are zero on all production and flow activities actually used and no activity may show a positive profit (conditions (8), (9) and (10),

2. Values of the hog and pork carcasses may exceed zero only if their regional net availability \((g_j)\) is equal to zero (condition (11) and (12)),

3. Rent on slaughter and processing plants may exceed zero only if the capacities are fully utilized (condition (13)),

4. Values on fixed demands must equal zero only if demands are fully met condition (14).

Thus, the optimality conditions specified by (8) through (14) yield values for the production and allocation problems that are consistent with the zero profit condition of competitive equilibrium.

In particular, condition (8) states that if any flow of the final products occurs between region i and j; i.e., \(q_{ij} > 0\), then \(-U_i^f - U_j^r = 0\). If \(U_i^f\) and \(U_j^r\) are interpreted as the values of the pork carcasses and the hogs at the demand and supply points respectively, and \(q_{ij} > 0\), then \(U_i^f - U_j^r = -t_{ij}\). Thus, the difference in value (rent) between demand and supply points i and j is the transportation cost.

Similarly, when no flows take place, \(q_{ij} = 0\), then \(U_i^f + U_j^r \leq t_{ij}\) and
the introduction of a flow from $i$ to $j$ would have the effect of increasing total transportation costs or increasing the rents paid. Because of condition (14), $U^+_i$, the rent or value of the pork carcass at the demand location, hog is transformed into a pork carcass in region $i$, that is, $\overline{A}_i > 0$, then the value of hog at the plant location, $\overline{U}^+_i P_i$ must be equal to the value of the hog at the supply point $U^+_i$ plus any internal rent that may accrue to the plant location, $U^+_i$ and $S_i$. Because of condition (12), $U^+_i$ is only positive when the supply of the hog in region $i$ is used to capacity. Because of condition (13), plant rent $U^+_i$ is positive only when the capacity of the plant is exhausted. Thus, the slaughtering plant location earns no rent and no rent or return is imported to the supply point unless capacity or supply is fully used.

Condition (10) states that if any hog flow from region $i$ to region $j$, that is, $Q_{ij} > 0$, then the difference between the value of the hog in the two regions is equal to the transportation cost, that is, $\overline{U}^+_i - \overline{U}^+_j = T_{ij}$. When no flows take place, $Q_{ij} = 0$, then $\overline{U}^+_i - \overline{U}^+_j \leq Q_{ij}$, a condition paralleling that for the final production or pork carcass.

4 The Dual Formulation and Interpretation

Given the Lagrangean (7) and the interpretation of conditions (8) through (14), equation (7) can be rewritten as follows:

\[ \mathcal{L}(X,U) = - \left( + \sum_i q_i U^+_i - \sum_i C_i U^+_i - \sum_i H_i U^+_i \right) \]

---

\[ + \sum_j q_{ij} (U_j^4 - U_i^4 - t_{ij}) + \sum_i A_i (P_i U_i^4 - U_i^4 - U_i^4) + P_i U_i^4 - S_i \]

\[ + \sum_i \sum_j Q_{ij} (U_j^4 - U_i^4 - T_{ij}) \]

Where 0 is the real number zero.

The Lagrangean (15) specifies the basis for the corresponding dual expression for the primal programming problem. If the system (1) through (6) is defined as the primal problem, then the algebraic expression of the dual programming problem is defined as:

To minimize:

(16) \[ G = - (\sum_i q_{i} U_i^4 - \sum_i C_i U_i^4 - \sum_i H_i M_i^4 - \sum_i O_i U_i^4) \]

Subject to

(17) \[ U_i^4 - U_i^4 - t_{ij} \leq 0 \]

(18) \[ P_i U_i^4 - U_i^4 - U_i^4 + P_i U_i^4 - S_i \leq 0 \]

(19) \[ U_j^4 - U_i^4 - T_{ij} \leq 0 \text{ for all } i \text{ and } j \]

(20) \[ U_i^4, U_i^4, U_i^4, U_i^4 \geq 0 \text{ for all } i \]

The dual programming problem is one of minimizing the total locational rents, subject to the conditions that (17), the difference between regional pork carcasses prices per unit, must be equal to or less than transportation cost, (18), the per unit value of the hog at the plant, must be equal to or less than the supply price plus rent accruing to plant location and (19), the regional difference between hog prices, must be equal to or less than the transportation costs. The restrictions of the dual programming problem are reflected in conditions (8) through (14) which state the condition that profits must be zero on all production and flow activities used and that no activity may permit a positive profit. Value or rent accrues to a supply or plant location only if the plant capacity and supply are fully used. In summary, the Lagrangean (15) and the \( u \) variable of the dual programming formulation (16) are interpreted as market.
values at each demand and supply point for the hog and pork carcasses.

5. The Programming Tableau

The tableau given in Table 1 shows the programming characteristics of the model specified by equations (1) through (7).

Table 1. A Two-Region Example of the Programming Tableau

<table>
<thead>
<tr>
<th>Internal prices and rents</th>
<th>Carcass Flows</th>
<th>Slaughtering hogs</th>
<th>Hog Flows</th>
<th>Regional constant</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$X_{11}$ 1 $X_{12}$ 1 $X_{21}$ 1 $X_{22}$</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>$Q_{12}$</td>
</tr>
<tr>
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<td>-P</td>
<td></td>
<td>= 0</td>
</tr>
<tr>
<td>$U_2$</td>
<td>1 1</td>
<td>-P</td>
<td></td>
<td>= 0</td>
</tr>
<tr>
<td>$U_3$</td>
<td>-1 -1</td>
<td></td>
<td></td>
<td>≤ 0</td>
</tr>
<tr>
<td>$U_4$</td>
<td>-1 -1</td>
<td></td>
<td></td>
<td>≤ 0</td>
</tr>
<tr>
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<td>-1 -1</td>
<td>≤ $Q_1$</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>≤ $Q_1$</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>≤ $C_1$</td>
<td></td>
</tr>
<tr>
<td>$U_{14}$</td>
<td>1</td>
<td></td>
<td>≤ $C_1$</td>
<td></td>
</tr>
</tbody>
</table>

Unit Costs

<table>
<thead>
<tr>
<th>$T_{11}$ $T_{12}$ $T_{21}$ $T_{22}$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$T_{12}$</th>
<th>$T_{21}$</th>
</tr>
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</table>

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