THE HOG CYCLE AS A HARMONIC MOTION IN TAIWAN?

Tso-Kwei Peng*

I. Introduction

The hog cycle was one of the earliest recognized economic cycles in Taiwan. A cycle is a pattern that some variables follow which repeats itself regularly over a period of time. A true cycle is self-energizing and not the result of chance factors. The length of a cycle is the time from one peak to the next or from one trough to the next, and is usually related to the time required to produce a new generation (Tomek and Robinson, 1972). The time lags involved in the hog production process and the interrelationships among price, pig crop, and slaughter contribute to the development of the cycle.

There are two ways to analyze cyclical supply hog production. One is based on the cobweb model; the usefulness of this approach has been confirmed in the works of Waugh (1964), Nerlove (1958), and Harlow (1962). Another approach is based on different forms of harmonic models. Both models explain commodity supply and require careful interpretation of a time-series graph of the data. Once an appropriate period is selected, the right length of expectation of farmers can be fitted in a cobweb model and a harmonic model. These two models can be estimated using regression methods to determine the amplitudes of the cyclical components.

The cobweb model is perhaps the classic illustration of a recursive system. The simplest cobweb model, provided by Ezekiel

* The author is Senior Specialist, Council for Agricultural Planning and Development, Executive Yuan, Taipei, Republic of China
in 1938, assumes that: (1) producers are price "takers" and supply response is based on price; (2) a clear time lag exists between a price change and a production change; (3) the total quantity planned to be produced is realized; (4) the quantity supplied in time t is sold in t, determining price in t; and (5) the supply and demand functions are linear and do not shift.

Some of the assumptions of the elementary cobweb model are not particularly critical and can be modified in a more realistic model. For instance, expected price may not be closely tied to the immediate past price, some products in the analytical commodity maybe in a less than competitive market, and realized production does not always equal planned production. Therefore, it is unrealistic to expect a perfect cycle with a constant period. Moreover, the simple cobweb model suggests that the hog cycle is twice length of the apparent lag (Dean and Heady, 1959). The hog cycle has typically been about four times the production period in U. S.

The simple cobweb model is clearly too elementary for application to most real world situations. A more detailed econometric model is necessary for significant applied analysis. Harlow (1962) used six equations to describe the supply, marketing, price formation and demand process which allows for shifts in the functions. To minimize interaction between price and quantity within a given period, Harlow used quarterly data in fitting a recursive system by the ordinary least squares.

The cobweb model can be modified into a conceptually different model. Larson (1964) has presented a harmonic motion model, which assumed that a cycle arises from producers maximizing profit with a linear demand function which includes price and the rate of price as variables. The model contains many of the features of the cobweb, but it modified the static short-run supply function and hence eliminates the cobweb. Larson's model of the hog cycle, like the cobweb model, relies on the assumptions of prices determined by current production and a fixed lag between the forming of producer's plans and the plan's realiza-
tion, producers plan output changes on the basis of current price, with the "strength of response... proportional to deviation from (long-run) equilibrium" (Larson, 1964). The harmonic motion model leads to price cycles of length equal to four times the fixed production lag, instead of twice the length of the as in cobweb model.

With the harmonic analysis technique, Kulshreshtha and Wilson (1973) analyzed the cattle and hog cycles in Canada and found that a 48 month cycle exists in hog prices, while no similar pattern in terms of a period of harmonics could be established in the hog slaughter number.

The objective of this paper is to evaluate the applicability of a harmonic motion model to explain the hog cycle in Taiwan.

II. Model

According to Abel (1962), this method has several advantages over alternative ways of analyzing cyclical fluctuations. First, the analysis permits one to work with the original data series rather than moving averages; second, the data can be compared against a fixed pattern of variation, and criteria for goodness of fit can be applied; third, its use permits one to test for changes in the pattern of seasonal variation over time, e.g., changes in amplitude or phasing; fourth, the seasonal pattern and trend, if it exists, can be estimated at the same time.

A typical economic time series can be decomposed into four major components: trend, seasonal variation, cyclical variation, and irregular variation. When trend is removed, a time series exhibits an almost cyclical appearance due to the presence of irregular variations in the residual $\zeta_t$. When the cyclical movements in a series cover the same time span, a sine-cosine curve can be fitted.

A harmonic function can be written as

$$Y_t = A \cos \frac{2\pi t}{T} + B \sin \frac{2\pi t}{T} \quad (1)$$

An expression which may also be written as
\[ Y_t = \sqrt{A^2 + B^2} \cos \left( \frac{2 \pi t}{T} - w \right) \] \hspace{1cm} (2)

Where \( w = \arctan \frac{B}{A} \). The value \( T \) is called the period of the harmonic, the reciprocal, \( 1/T \), the frequency, the quantity, \( \sqrt{A^2 + B^2} \), the amplitude, and \( w \) the phase angle. Sometimes, \( A \) and \( B \) are referred to as the components of the harmonic (Davis, 1941).

Equation (1) can be fitted by the method of least squares and the form of equation becomes:

\[ T_t = K + A \cos \frac{2 \pi t}{T} + B \sin \frac{2 \pi t}{T} + U_t \] \hspace{1cm} (3)

Where \( U_t \) is random term with mean zero and constant variance, \( \sigma_u^2 \). If the basic assumptions of applying the ordinary least squares are met, then, the estimator of OLS is a BLUE. The constant term \( K \) appears in equation (3) because moments about the mean are used to compute the regression coefficients. In general, it will not be significantly different from zero (Abel, 1962). A trend variable, \( t \), can be included in (3) so that

\[ Y_t = K + (A + a_t) \cos \left( \frac{2 \pi t}{T} + w \right) + (B + b_t) \sin \left( \frac{2 \pi t}{T} + w \right) + U_t \] \hspace{1cm} (4)

A more general application of the harmonic model has been rationalized by Abel (1962). He indicated that the model can test whether the amplitude changed over time, and whether there has been a shift in cyclical pattern. The model is:

\[ Y_t = K + (A + a_t) \cos \left( \frac{2 \pi t}{T} + w \right) + (B + b_t) \sin \left( \frac{2 \pi t}{T} + w \right) + U_t \] \hspace{1cm} (5)

Linear trend terms, \( a_t \) and \( b_t \), are included in the amplitude coefficient which determines the rate of change in amplitude over time and is a phase angle used to reveal movements in cyclical patterns such as a shifting seasonal (Labya, 1973).

Harmonic analysis can be used as a measure of cyclical behavior in economic series by criteria of goodness of fit. Or one
can use the model to explain commodity supply. For the latter use, it requires careful interpretation of the time-series graph of the data. Once an appropriate period is selected, the equation in its simpler form can be estimated to determine the amplitudes of cyclical components, \( a_t \) and \( b_t \). If the possibility exists of a trend in amplitude, equation (5) can be estimated where the coefficients are expanded (Labys, 1973).

III. Empirical Results

By applying spectral analysis using monthly data from 1950 to 1973, the cycle length of hog production in Taiwan was shown to be 44 months (Ko, 1976). Also, from Fourier Series Analysis, two minor cyclical fluctuations with lengths of 22 months and 11 months were identified. Hog production in Taiwan is a multi-frequency cobweb model developed by Talpaz (1974). According to the data of Taiwan Provincial Government, the whole process of hog production from breeding to slaughter requires 11 months and for giltssaved for breeding, another 11 months are needed from breeding to slaughter weight (usually 95 kg. per her head). Therefore, a long-run hog raising plan in Taiwan needs 22 months. As a result, a complete cycle requires 44 months. Besides long run cyclical variation, there are intermediate and short run cyclical variations. These minor cyclical movements may be interpreted as adjustments under fixed facility and technology.

On the basis of Ko's results, the cyclical component was defined as having a period of 44 months. The seasonal components, of course, have a period of 12 months. Therefore, a tentative model for hog price or quantity can be defined as follows:

\[
Y_t = a_1 + a_2 T + a_3 \cos \frac{2 \pi t}{44} + a_4 \sin \frac{2 \pi t}{44} + a_5 \cos \frac{2 \pi t}{12} + a_6 \sin \frac{2 \pi t}{12} + u_t 
\]

where

\( T = \text{trend} \)
\[ a_3 \cos \frac{2 \pi t}{44} + a_4 \sin \frac{2 \pi t}{44} = \text{the cyclical variation} \]

\[ a_5 \cos \frac{2 \pi t}{12} + a_6 \sin \frac{2 \pi t}{12} = \text{the seasonal variation} \]

By using OLS with monthly data for hog prices and slaughter numbers from January 1967 to December 1979, the estimated results of the harmonic motion model for these two time series are:

1. Price of hogs (PH)
   \[ PH_t = 15348 + 21.856 T + 8.749 \cos \frac{2 \pi t}{44} - 46.356 \sin \frac{2 \pi t}{44} \]
   \[ (15.243) \quad (19.644) \quad (0.123) \quad (-0.656) \]
   \[ -16.45 \cos \frac{2 \pi t}{12} + 9.051 \sin \frac{2 \pi t}{12} \]
   \[ (-0.232) \quad (0.127) \]
   \[ R^2 = 0.72 \quad \bar{R}^2 = 0.71 \quad d = 0.165 \]

2. Slaughter number of hogs (H)
   \[ H_t = 258530 + 1919.0 T + 1974.1 \cos \frac{2 \pi t}{44} + 5445.7 \sin \frac{2 \pi t}{44} \]
   \[ (28.119) \quad (18.887) \quad (0.305) \quad (0.844) \]
   \[ -1832.0 \cos \frac{2 \pi t}{12} - 6671.3 \sin \frac{2 \pi t}{12} \]
   \[ (-0.283) \quad (-1.033) \]
   \[ R^2 = 0.71 \quad \bar{R}^2 = 0.70 \quad d = 0.942 \]

In equations (7) and (8), none of the coefficients differs significantly from zero according to t-test except the intercept terms and the trend variables. Thus, we may conclude that the length of hog cycle in Taiwan is not 44 months any more.

The different lengths of the cyclical component have been estimated using the contribution criteria as suggested by Doran and Quilkey (1972). With length varying from 48, 36, 33, 24, and 22 months, none of them revealed the periodic time span which can be fitted. Logarithmic form for slaughter number and hog prices combined with different length of cycles also proved that the hog cycle in Taiwan did not exist a fixed time span. As a result, we might conclude that the hog cycle in Taiwan does not exist with a regular undulating oscillation. Also, neither is there a consistent seasonal variation.
IV. Discussion

The crucial assumption of the harmonic motion model is that it involves continuous variables and includes variables measured as deviations from a long-run equilibrium (Hartman, 1974). In this regard, the harmonic motion model is difficult to use. Hog production in Taiwan is of a small scale, and farmers usually do not produce hogs continuously, so that, the basic assumption is violated. Moreover, government regulation has substantially influenced hog production. Waugh (1964) claims that price policy can interrupt the cycle. As a result, the exogenous variables have probably disturbed the natural cycle in hog production in Taiwan so that there is no significant hog cycle period. While the $R^2$'s value are reasonably good, must be due to the trend.

From the mathematic standpoint, according to the Wold Theorem, any stationary process series can be represented as the sum of two mutually uncorrelated processes, that is, linearly deterministic and purely nondeterministic. For instance, a classical model for time series data is

$$X_t = T(t) + S(t) + C(t) + Y_t$$  \hspace{1cm} (9)

where $T(t)$ is a deterministic component representing the trend, the seasonal $S(t)$ is also deterministic and is a period component with 12 months, $C(t)$ is a cycle component, and $Y_t$ is a stationary process with no deterministic components (Granger and Newbold, 1977). In fact, there is no evidence that hog production contains periodic components other than seasonal ones, so the deterministic harmonic model cannot explain the variance of variables. As a result, time series analysis with a stochastic process assumption on the time series data is needed to explain the hog cycle in Taiwan.

Footnotes

1. Larson's model of the hog industry is:
(1) linear demand curve with price dependent on current output only, i.e.,
\[ P_t = a - b Q_t \]

(2) fixed production process with a 12 month lag between sow bred (B) and production (Q)
\[ Q_{t+12} = c B_t \]

(3) the rate of change of breeding is proportional to the deviation of price from equilibrium, i.e.,
\[ \frac{dB_t}{dt} = e(P_t - P) \]

2. If the trend component is removed from the series, the properties of estimators obtained from OLS is different from the estimators obtained from the original data. Therefore, the test of significance of t and F ratios can not be applied. For details, see Doran and Quilkey (1972).

3. The general long-run hog raising plan in Taiwan can be described as follows:

4 months 7 months
sow \rightarrow piglet \rightarrowweaning & feeding \rightarrowslaughter hog
↑ pregnancy breeding (gilt or barrows)

References


