CONCEPTUAL MODEL FOR U. S. CORN AND SOYBEAN CASH AND FUTURES MARKETS

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The purpose of this paper is to survey and present a brief synopsis of the theoretical concepts that forms the basis for the econometric model of U.S. corn and soybean sectors. The focus is on theory relevant to the econometric estimation of demand and supply response, and on the theory of price-of-storage. The theory of simultaneous equilibrium in cash, futures and storage markets is then developed.

I. DEMAND THEORY

The estimation of demand is of critical importance in modeling historical price movements. The three kinds of demand important in corn and soybean markets to be considered explicitly are consumer demand for food uses, derived demand for feed fed to livestock, (demand for soybeans is derived from the demand for soybean meal and soybean oil, in turn, from livestock and consumer demands), and export and import demand by the foreign sector. The theory of consumer behavior accounts for final demand and the theory of the firm gives rise to derived demand. The total demand for commodities is also influenced by the demand for inventories. In this section, we will attempt to analyze the aspects of demand theory that can be used to improve the empirical specification of commodity demand relationships. The theoretical basis for this model is a variant of the competitive model of price determination.

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Consumption demand

The relationships used to explain food demand for corn and soybean products are derived from consumer demand theory. Demand theory is based on the maximization of a consumer's utility subject to an appropriate budget constraint. Solution of the maximization problem leads to a set of demand equations of the form:

$$q_{it}^{d} = F_i (P_{it}, P_{jt}, Y_t, \underset{\sim}{K}_t) \tag{1}$$

which relates consumption of commodity i, q_{it}^{d}, to its price, P_{it}, the price of other commodities, P_{jt}, income, Y_t, and other variables that influence consumer taste and preferences, $\underset{\sim}{K}_t$.

Under the assumption of identical consumer preferences, total market demand Q_{it}^{d} can be written as the product of population (POP) and per capita demands:

$$Q_{it}^{d} = POP \cdot F_i (P_{it}, P_{jt}, Y_t, \underset{\sim}{K}_t) \tag{2}$$

The assumption of the representative consumer or identical preferences leads to unitary population elasticities[1]. Thus, the equation (2) can be rewritten in per capita terms as

$$\frac{Q_{it}^{d}}{POP} = F_1 (P_{it}, P_{jt}, Y_t, \underset{\sim}{K}_t) \qquad (3) \qquad (3)$$

Constrained maximization of utility results in a set of underlying restrictions known as the Engel aggregation, Cournot aggregation, homogeneity, symmetry and Hicks-Slutsky conditions.[2]

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[1]Empirically, this specification which restricts the population elasticity to be one, thereby avoids multicollinearity between population and income.

[2]More details see Labys (1973, p. 18).

In the absence of externalities, a market demand relation can be thought of as the summation of individual demand relations. The static concept of demand refers to movements along the multidimensional demand function. Dynamic adjustments in consumer demand to change in demand variables can be incorporated through habit persistence, adaptive expectations or partial adjustment formulations (Nerlove, 1958).

Derived demand

The term "derived demand" is used to denote demand schedules for inputs which are used to produce the final products. Demand for corn for feeding to livestock and demand for soybeans for crushing into oil for cooking or salad oil and meal for livestock feeding are examples of derived demand. The theoretical derivation of factor demand and its properties are well-documented throughout the literature (R.G.D. Allen (1956), Baumol (1977), Henderson and Quandt (1980)).

Consider derived demand for corn and soybean meal by the livestock sector. Assume the existence of an implicit production relationship in which feed grains, Q_f, are employed along with unfattened feeder animals, D_{fr}, to produce fattened animals, A, such that

$$q\,(A, Q_f, D_{fr}, \underset{\sim}{Z}) = 0 \tag{4}$$

In this case, the livestock farmer's feed grain demands are derived from the underlying demand for the livestock which they produce. The producer is assumed to maximize his porfits subject to constraints as specified by production relationships and market conditions.

The optimization procedures for profit maximization by the farmer starts by assuming:

A profit function,

$$\pi = P_1\,A - P_f Q_f - P_{fr} D_{fr} \tag{5}$$

which is to be maximized subject to the production relationships

$$q\ (A, Q_f, D_{fr}, \underset{\sim}{Z}) = 0 \tag{6}$$

and market demands for livestock

$$A = h\ (P_l, P_{ls}, P_o, \underset{\sim}{Z}) \tag{7}$$

where P_l = price of slaughter animals,
P_{ls} = price of livestock product,
P_o = other prices affecting demand for livestock,
P_f = price of feed grains,
P_{fr} = price of feeder animals,
$\underset{\sim}{Z}$ = other influencing factors.

The problem above can be reformulated by using the Lagrangian function

$$\begin{aligned} &\text{Max } L\ (A, Q_f, D_{fr}, \gamma\ ,\ \delta) \\ &= P_l A - P_f Q_f - P_{fr} D_{fr} + \gamma\ [0 - q\ (A, Q_f, D_{fr}, \underset{\sim}{Z})] \\ &+ \delta\ [A - h\ (P_l, P_{ls}, P_o, \underset{\sim}{Z})] \end{aligned}$$

where r is a Lagrangian multiplier and is equal to the marginal profit to the representative livestock farmer due to an exogenous shift in the production relationship. δ is another Lagrangian multiplier and is equal to the marginal profit to the representative livestock farmer due to an exogenous shift in demand for livestock.

The first order conditions for the maximization of (8) are:

$$\frac{\partial L}{\partial A} = P_l - r\frac{\partial q}{\partial A} + \delta = 0 \tag{8a}$$

$$\frac{\partial L}{\partial Q_f} = -P_f - r\frac{\partial q}{\partial Q_f} = 0 \tag{8b}$$

$$\frac{\partial L}{\partial D_{fr}} = -P_{fr} - r\frac{\partial q}{\partial D_{fr}} = 0 \tag{8c}$$

$$\frac{\partial L}{\partial \gamma} = g\,(\,A\,,\,Q_f\,,\,D_{fr}\,,\,\underset{\sim}{Z}\,) = 0 \qquad (8d)$$

$$\frac{\partial L}{\partial \delta} = A - h\,(\,P_l\,,\,P_{ls}\,,\,P_o\,,\,\underset{\sim}{Z}\,) = 0 \qquad (8e)$$

Solving the equations simultaneously[1] gives the optimal quantities of A^*, O_f, D_{fr}, r^* and δ^*:

$$A^* = A\,(P_l, P_{ls}, P_{fr}, P_f, P_o, \underset{\sim}{Z}) \qquad (9a)$$

$$Q_f^* = Q_f\,(P_l, P_{ls}, P_{fr}, P_f, P_o, \underset{\sim}{Z}) \qquad (9b)$$

$$D_{fr}^* = D_{fr}\,(P_l, P_{ls}, P_{fr}, P_f, P_o, \underset{\sim}{Z}) \qquad (9c)$$

$$r^* = r\,(P_l, P_{ls}, P_{fr}, P_f, P_o, \underset{\sim}{Z}) \qquad (9d)$$

$$\delta^* = \;(P_l, P_{ls}, P_{fr}, P_f, P_o, \underset{\sim}{Z}) \qquad (9e)$$

Function (9b) expresses the derived demand function for feed and provides the feed demand relationships for estimation purposes.

The market demand curve for feed grain is obtained by aggregating the individual livestock farmer's feed grain input demand curves. However, the market demand cannot be obtained by simply adding together each farmer's demand curve because as the price of feed falls and more of it is used, output of livestock will increase. As the output of the entire industry increases, the price of the product will fall. This results in a shift in individual feed demand curves. This reduction in the

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[1]The second order conditions which must hold for this to be a point of profit maximization, require that the principal minors of the relevant Hessian determinant alternate in sign (see Henderson and Quandt, 1980, p. 96). Meeting this sufficiency condition implies that the marginal physical products for all inputs in all alternative uses must be diminishing.

price of livestock output in deriving the market demand curve for feed grain must be accounted for in deriving the market demand for feed grain.

In Figure 1, a decline in the price of feed grain from P_f^o to P_f^1 will involve a movement down the Q_f^o crve to Q_1, given the following three assumptions:

1. The price of livestock is constant.

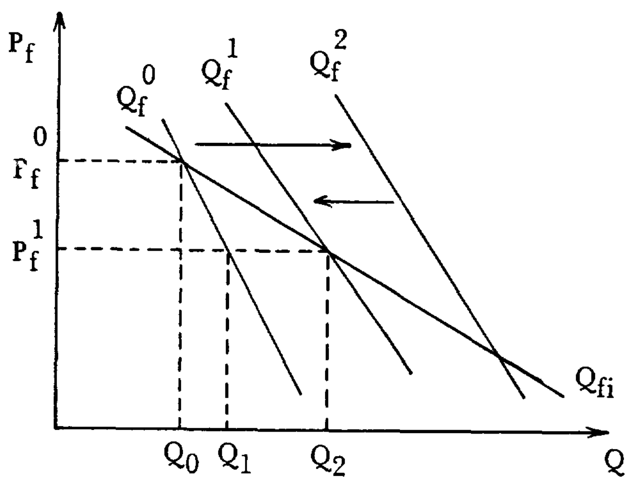

Figure 1. Derivation of the market level Demand for feed grains

2. The price of other inputs and quantities employed remain unaltered.

3. The technical input-output relationship remains the same. However, increases in the feed grains used cause the marginal product of substitute inputs to decrease and the marginal product of complementary input to increase, thereby shifting demand from Q_f^o to Q_f^2. Meanwhile, the quantity of livestock produced will increase as all producers utilize more of the lower priced feed grains. This will cause the output price to fall with a resulting leftward shift of feed demand from Q_f^2 to Q_f^1. Hence, the derived demand curve for feed grains is Q_{fi} for any representative farmer i. The market derived demand curve Q_f^m is the summation of all farmers derived demand curves, i.e.,

$$Q_f^m = \sum_{i=1}^{n} Q_{fi}$$

This curve shifts in response to changes in prices of other inputs as well as technological change. The derived demand function may include other possible shifters:

$$Q_f^m = f\,(P_l, P_{lp}, P_{fr}, P_f, \#\,LS, P_{oi}, T, Z) \qquad (10)$$

where Pl, P_{lp}, P_{fr}, P_f, Z the same definition as described on page 4, # LS is the number of livestock, P_{oi} is the price of other feed input, and T is technological change.

In summary, the demand for feed grain depends on conditions in the final product market, supply conditions for other inputs and the production relationships between inputs and outputs.

Import/export demand

The demand for U.S. exports is derived from the supply and demand conditions of the rest of the world (ROW). The traditional theory of import demand is based on the proposition that import and domestic goods are not perfect substitutes (Leamer and Stern, 1970, p. 11). The quantity of imports purchased by any consumer in importing countries will depend on income, the price of imports and the price of all other goods. The specification is analogous to conventional demand analysis given by the following equation:

$$M = f(P_m, P_o, Y) \qquad (11)$$

where M = average import demand

P_m = price of import good

P_o = price of all other goods

Y = the level of income in the importing countries

If imports and domestic goods are perfect substitutes as is the case for both corn and soybeans, then under the assumption of perfect competition, the import demand for foreign goods is an excess demand, and prices are spatially competitive.

Worldwide equilibrium price and quantity determination for perfectly competitive goods can be illustrated with a generalization of a two region market. Each region has known supply and demand functions and produces and consumes a homogenous product. The regions are separated but not isolated by known transfer costs. Given this knowledge, the problem is to determine the equilibrium levels of production, consumption and prices in each region and the equilibrium trade flows between regions.

In Figure 2a and 2b, region 1 is considered to be an exporting country (the U.S. which exports corn and soybeans) and region 2 is considered to be all importing countries (Japan, the EEC, etc., which import corn and soybeans). In the model presented in Figure 2a, transfer costs are assumed to equal zero. The first and third graphs of Figure 2a contain the known supply and demand function (D_1, S_1, D_2, S_2) for regions 1 and 2. If trade was not allowed, each region would reach equilibrium at the intersection of its supply and demand function (i.e., $\hat{P}_1$, $\hat{Y}_1$, $\hat{X}_1$, $\hat{P}_2$, $\hat{Y}_2$, $\hat{X}_2$). However, if free trade is allow-

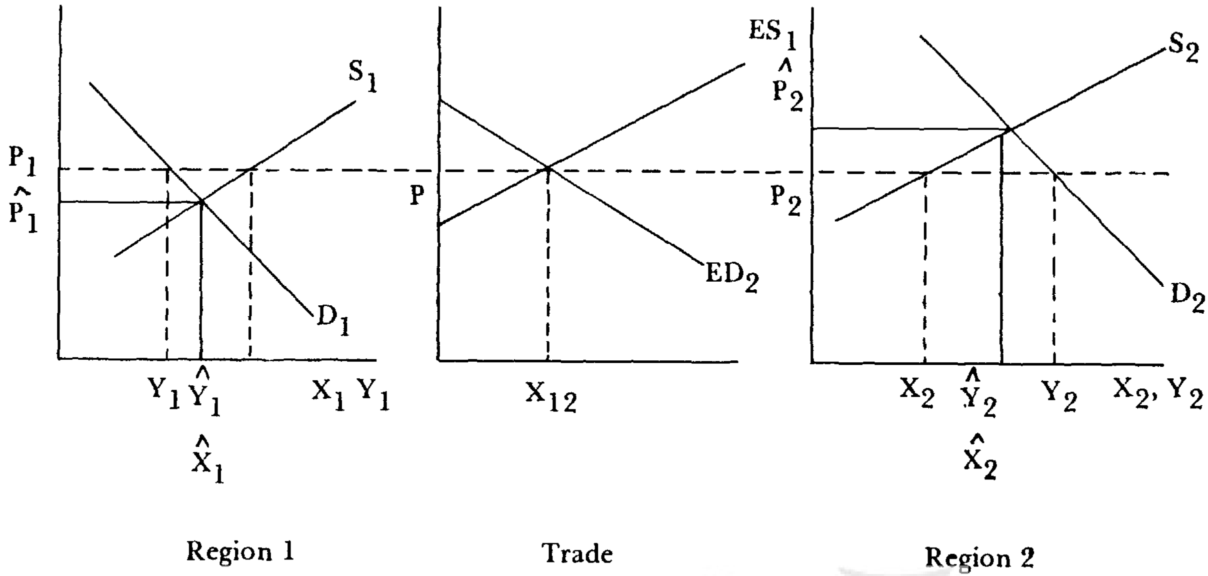

Figure 2a. Free trade model without transfer cost and exchange rate effects

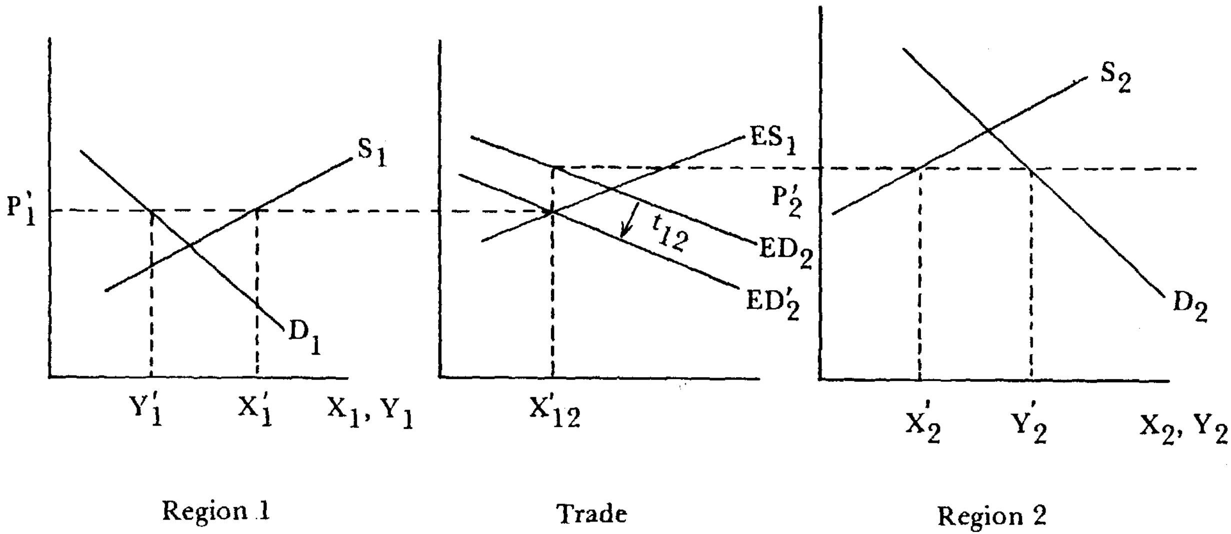

Figure 2b. Free trade model with transfer cost and exchange effects

ed, traders would recognize the opportunity to profit by arbitraging product from region 1 to region 2. The quantity offered for trade by region 1 is the difference between the quantity producers supply and that which consumers demand at prices higher than $\hat{P}_1$. This is described as the excess supply function (ES_1), which is plotted in the center graph.

The quantity region 2 offers to purchase is the difference between the quantity consumers demand and producers supply at prices lower than $\hat{P}_2$. This is described as the excess demand function (ED_2) in the center graph.

Equilibrium is reached at the intersection of ED_2 and ES_1. Price is equal in both markets, equilibrium production and consumption are X_1, Y_1 for region 1 and X_2, Y_2 for region 2, and the equilibrium level of trade is X_{12}. The quantity traded between the two regions is equal to the differences between the quantities supplied and demanded within each region, i.e., $(X_1 - Y_1) = X_{12} = (Y_2 - X_2)$. Also, at this equilibrium, $P_1 = P_2 = P$.

In Figure 2b, the two region model is reproduced, but a transfer cost of t_{12}, is added to reflect the transportation cost. The effect of changes in transportation costs is to shift the excess demand schedule faced by importing countries. Equilibrium is accomplished by shifting ED_2 downward by t_{12} since it represents an additional cost to ED_2'. Equilibrium is reached for region 2 at P_2', X_2' and Y_2' and for region 1 at P_1', X_1', Y_1'. If the exchange rate effect is also taken into account, the only change in the equilibrium condition is that:

$$P_1' + t_{12} = eP_2' \tag{12}$$

where e is the exchange rate which is evaluated in terms of the unit of the exporter's currency per unit of the importer's currency. This price relationship will be discussed in more detail in the following section.

Based on the above theoretical framework, the spatial equilibrium market model can be constructed.

$$\text{Exporting Country}\begin{cases}\text{Supply} & S_i = S_i(P_i, \underset{\sim}{X}) & (13a)\\ \text{Demand} & D_i = D_i(P_i, \underset{\sim}{X}) & (13b)\\ \text{Excess Supply} & ES_i = S_i(P_i, \underset{\sim}{X}) - D_i(P_i, \underset{\sim}{X}) = ES_i(P_i, \underset{\sim}{X}) & (13c)\end{cases}$$

$$\text{Importing Country}\begin{cases}\text{Supply} & S_j = S_j(P_j, \underset{\sim}{X}) & (13d)\\ \text{Demand} & D_j = D_j(P_j, \underset{\sim}{X}) & (13e)\\ \text{Excess Demand} & ED_j = D_j(P_j, \underset{\sim}{X}) - S_j(P_j, \underset{\sim}{X}) = ED_j(P_j, \underset{\sim}{X}) & (13f)\end{cases}$$

Price-linkage $$P_i + t_{ij} = eP_j \text{ or } \frac{P_i + t_{ij}}{e} = P_j \qquad (13g)$$

Spatial Equilibrium $$\Sigma_i ES_i = \Sigma_j ED_j \qquad (13h)$$

where S_i = supply of corn (soybeans) in exporting country i,
D_i = demand of corn (soybeans) in exporting country i,
P_i = price of corn (soybeans) which related to demand and suply in country i,
ES_i = excess supply of corn (soybeans) in country i,
S_j = supply of corn (soybeans) in importing country j,
D_j = demand of corn (soybeans) in importing country j,
P_j = price of corn (soybeans) which related to demand and supply in country j,
ED_j = exchange demand of corn (soybeans) in country j,
e = exchange rate evaluated in terms of the units of the exporter's currency per unit of the importer's currency,
t_{ij} = transportation cost between country i and j,
$\underset{\sim}{X}$ = other exogenous variables.

In the above spatial model, equilibrium occurs where $\sum_j ED_j = \sum_i ES_i$. The price-linkage between countries i and j is through transportation costs (t_{ij}) and the exchange rate (e) as shown in equation (13g). Trade theory suggests that importers view international prices in terms of their own currency. This captures the exchange rate effect.

The market model consists of the eight equations, (13a) through (13h), and the eight endogenous variables, S_i, D_i, S_j, D_j, ES_i, ED_j, P_i and P_j. Substitutions can be made to obtain an expression for the import demand equation

$$ED_j = f\left(\frac{P_i + t_{ij}}{e}, \underset{\sim}{X}\right) \tag{14}$$

Excess demand for corn (soybeans) is a demand derived from the importing country's derived demand for corn (soybeans). Thus, import demand for corn (soybeans) is affected by the price of livestock P_{jl}, the price of substitute feed P_{jf}, the number of livestock $\# LS_j$, and income levels Y_j in the importing country j. Corn (soybean) supply in importing countries Q_I and other competing supply of corn (soybeans) S_o would also affect U.S. corn (soybean) exports. The above function (14) is further modified to incorporate those factors above, so that world demand for U.S. exports of corn (soybeans) can be expressed as

$$ED_j = f\left(\frac{P_i + t_{ij}}{e_{ij}}, P_{jf}, P_{jl}, \# LS_j, Y_j, Q_I, S_o, \underset{\sim}{X}\right) \tag{15}$$

Theory of Supply Response

One of the objectives of this study is to examine the role of futures prices in production decisions and supply response for both corn and soybean producers.

In this section, the theoretical framework of dynamic supply response is first considered. The emphasis is on the producer's behavior in both the short-run and the long-run in response to changes in price expectations. Then, the formulation of price expectations is discussed. Special consideration is given to the role that the futures prices, as market-determined observations of expected prices, might perform in shaping output decisions.

The nature of dynamic supply response

Unlike static theory, dynamic supply response introduces consideration of "time" into the supply function. A time lag within agricultural production exists so that producers do not make full adjustments within a discrete period but instead distribute their adjustments among future periods until they finally approach an optimal position.

In the dynamic supply response, there are long- and short-run adjustment periods. The following dynamic adjustment model allows the consideration of how producers react to price changes over time. The adjustment, in general, is affected by the producer's expectations and the information available for production decision-making. The following discussion of dynamic supply response is based on Nerlove's discussion (Nerlove, 1958).

Figure 3 assumes a once-and-for-all change in price or assumes that a farmer's long-run expected price is constant. S_1S_1 is the long-run supply curve. The point B on S_1S_1 represents an equilibrium of demand and supply. At a price of OA, the quantity of B is supplied each period. If the demand curve shifts so that the price is now OC, the quantity supplied does not increase immediately to CP, where P is a point on the long-run supply curve, but to CD, where D is a point on one of the short run supply curves through B. If the price remains at OC, we would observe the quantity CE supplied in the following period, then CF, then CG, CH, and so on. Each of the points D, E, F, G, H, lie on different short run supply curves through point B. As time passes, they gradually approach point P, which lies on the long-run supply curve.

In reality, price will be changing continuously. Hence, the

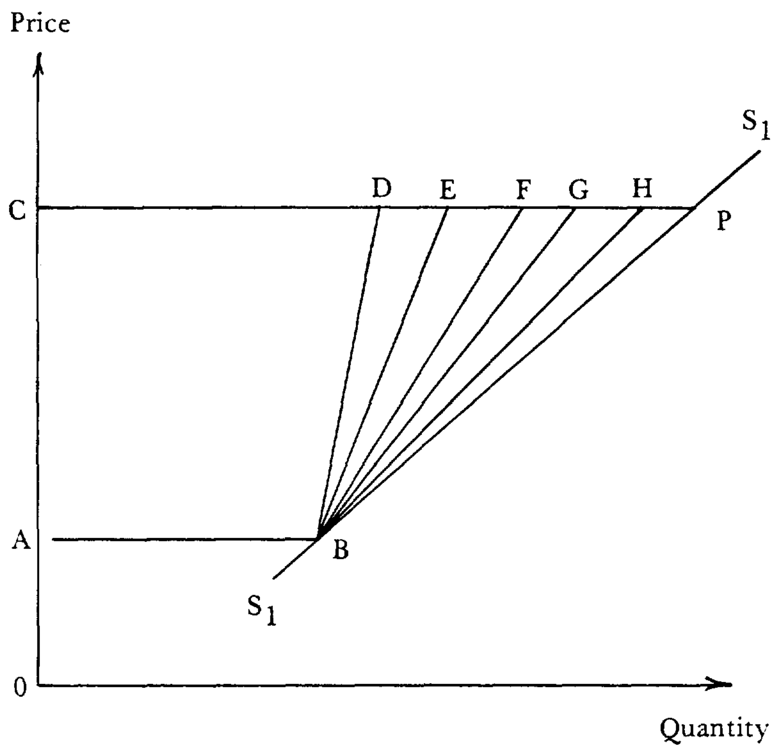

Figure 3. Adjustment of the quantity supplied to once-and-for-all change in price.

point we observe never lies on the long-run supply curve. In figure 4, we start out, as before, from an initial equilibrium point B on the long-run supply curve S_1S_1. Now let the demand shift in such a way that the price increases constantly, first to OC, then to OE, OG, OI, etc. When the price increases from OA to OC, producers adjust their output from AB to CD. If the price remained at OC, they would produce CW in the following period; but the price increases again to OE. Consequently, they move along a new short-run supply curve through the point W to F. Producers supply slightly more than they would have had the price remained at OC; thus, as the price increases, we observe a series of points, D, F, H, J, L, etc., which lie on short-run supply curves passing through different points on the long-run supply curve. S_eS_e is an approximization of the long-run supply curve which we want to estimate, but it is not the true long run supply curve S_1S_1. The short-run and long-run supply elasticities may be estimated only through S_eS_e, but not S_1S_1.

Formulation of price expectations

For the crops considered in this study, there is a five to six month period between planting and harvesting. As a result, producers do not know the harvest price when making planting decisions. When decision-making is undertaken with incomplete information about the future, producers base their decisions on prices they expect to prevail. The proper supply response model for this behavioral relationship should incorporate the price expectations and production adjustments. The general form of the model is:

$$A_t^e = a + b\,P_t^e + \sum_{i=1}^{n} c_i Z_{it} + U_t \qquad (16)$$

where A_t = Output or acreage planted at time t,

P_t^e = The expected price at time t formed at t-k,

Z_{it} = Other inputs or shifter variables at time t, i=1, . . , n.

U_t = The random disturbance terms.

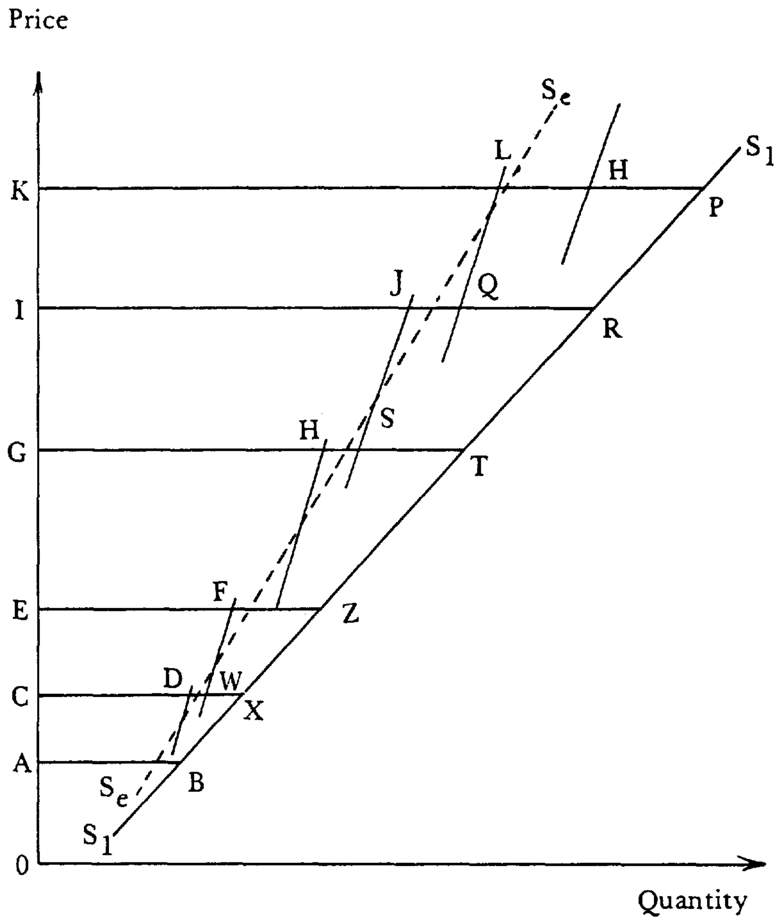

Figure 4. Adjustment of the quantity supplied to successive change in price.

Since P_t^e is unobservable, additional assumptions are necessary to relate this unobserved variable to variables which can actually be observed. The following hypothesized models attempt to do this.

a. Naive expectations (Cobweb Model): where

$$P_t^e = P_{t-1} \tag{17}$$

This model suggests that the current expected price is equal to the previous period's actual price (P_{t-1}).

b. Extrapolative expectations: where

$$\begin{aligned} P_t^e &= P_{t-1} - \alpha (P_{t-1} - P_{t-2}) \qquad 0 < \alpha < 1 \\ &= (1-\alpha) P_{t-1} + \alpha P_{t-2} \end{aligned} \tag{18}$$

The current expected price is a weighted combination of the two previously observed prices (P_{t-1}, P_{t-2}). This model takes into account the most recent trend in prices.

c. Adaptive expectations model: where

$$P_t^e = P_{t-1}^e + \beta (P_{t-1} - P_{t-1}^e) \qquad 0 < \beta \leq 1 \tag{19}$$

β = the rate of adaptation

This model expresses the current price expectation as the price expectation generated in the previous period (P_{t-1}^e), plus a proportion (β) of the previous period error ($P_{t-1} - P_{t-1}^e$). Solving the adaptive expectations difference equation yields

$$P_t^e = \beta \sum_{i=0}^{\infty} (1-\beta)^i P_{t-1-i} \tag{20}$$

Thus, the expectation is expressed as an infinite, geometrically weighted, moving average process.

d. Partial adjustment model: where

$$A_t^e = a + b\,P_{t-1} + \sum_{i=1}^{n} c_i Z_{it} + U \qquad (21)$$

A_t^e = desired supply

and

$$A_t = A_{t-1} + r\,(A_t^e - A_{t-1}) \qquad 0 < r \leq 1 \qquad (22)$$

r = the output adjustment coefficient

Solving the partial adjustment difference equation gives

$$A_t^e = r \sum_{i=0}^{\infty} (1-r)^i A_t \qquad (23)$$

e. Compound model: This model integrates models (c) and (d).

where

$$A_t^e = a + b\,P_t^e + \sum_{i=1}^{\infty} C_i Z_{it} + U_t \qquad (24)$$

$$A_t^e = r \sum_{i=1}^{\infty} (1-r)^i A_t$$

$$P_t^e = \beta \sum_{i=0}^{\infty} (1-\beta)^i P_{t-1-i}$$

The use of Nerlove's dynamic supply model showed marked improvements over static or naive models. However, the distributed lag approach has been criticized by Griliches (1967, p. 42) as being "theoretical ad-hockery." The intuitive argument against this approach is that it is "backward-looking", only contemplating the past behavior of prices. It ignores new information available after the last cash price was determined. One possibility for obtaining "forward-looking" expectations is to use futures prices as expected prices in the future. This approach is based on the rational expectation hypothesis (Muth, 1961). Expectations are based on information and generated according to perceptible forces affecting the economic activity to be investigated.

Futures price in supply analysis

Since the appropriate price for supply analysis is the price expected by producers when production decisions are being made, the futures price prevailing when production decisions are made makes a reasonable candidate for a directly observable measure of the expected product price in supply analysis. A theoretically well-grounded hypothesis[1] is that the price of a futures contract for the new crop reflects the market estimate of the new crop cash price.

$$P_t^e = E_{t-k}(P_t) = FP_{t-k}^{t}$$

or

$$P_{t+k}^e = E_t(P_{t+k}) = FP_t^{t+k} \qquad (25)$$

where FP_{t-k}^{t} (or FP_t^{t+k}) is the price at time t-k (or t) quote for futures deliverable in period t (or t+k).

The use of futures prices as an observation of expected crop prices raises two issues of concern. First, the market's estimate as given by a futures price reflects the expectations of nonfarm speculators as well as crop producers. Thus, do farmers use information efficiently if they have access to the same informa-

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[1]The extent to which futures prices are unbiased estimates of subsequent cash prices has been explored in a large number of theoretical and empirical studies, e.g., Working (1942), Brennan (1958), Telser (1958), Tomek and Gray (1971), Peck (1976), Gardner (1976). In each of these cases, the hypothesis was maintained that futures prices provide an exact measure of the price expectations. Their conclusions from this empirical analysis, especially for storable commodities, was that the futures price can be considered an unbiased prediction of the subsequent spot price.

tion available to the informed speculators acting in futures markets? Second, which futures price is most appropriate and at what date this price become producers' expected price?

Both of the above issues can be answered by Muth's theory of rational expectations (Muth, 1961). Muth provided a link between survey studies of expectation and econometric approaches.

The rational expectations theory is based on the following hypothesis about individual behavior (Muth 1961, p. 316),

> ". . . information is scarce, and the economic system generally does not waste it; . . . the way expectations are formed depends specifically on the structure of the relevant system describing the economy; . . ."

In order to derive the price expected to prevail at period t (P_t^e), on the basis of information through t-k periods, Muth (1961, p. 333) further assumed the actual and expected price relationships to be

$$P_t = P_t^e + V_t \tag{26}$$

where V_t is error in period t and the structure of V_t is assumed to be known, and

$$E(V_t) = 0$$

$$E(P_t^e V_t) = 0$$

If the expectations which are formed to predict future events are the same as the predictions of relevant economic theory, then these expectations are rational. Mathematically, the expected value of the market price at time t equals the market price expected to prevail at time t on the basis of information available at time t-k or,

$$E_{t-k}\,(P_t/I_{t-k}) = P_t^e \qquad (27)$$

where I_{t-k} is information available in period t-k.

Incorporating equation (27) into his model[1] gives

$$\begin{aligned}
&\text{Demand:} && D_t = -b\,P_t \\
&\text{Supply:} && S_t = c\,P_t^e + U_t \qquad (28)\\
&\text{Market equilibrium:} && D_t = S_t \\
&\text{Expectation function:} && P_t^e = E_{t-k}\,(P_t)
\end{aligned}$$

where D_t = quantity demanded in period t,

S_t = quantity supplied in period t,

P_t = the market price in period t,

P_t^e = the price expected to prevail in period t on the basis of information through period t-k,

U_t = stochastic error.

According to the rational expectations hypothesis, the price expectations in this model are unbiased and the expected price is treated as endogenous to the system.[2]

Given rational expectations, we can answer the first issue of the use of futures prices as indicated previously. This theory implies that economic behavior underlies the formulation of expectations. Expectations are based on information and generated according to perceptible forces affecting the economic activity to be investigated. Under this hypothesis, if a producer operating under free competition has some idea of

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[1]All variables used are deviations from equilibrium values in his model.

[2]D_t, S_t, P_t and P_t are endogenous to this system. The single expression of the expected price (P_t^e) is solved by system (28) as

$$P_t^e = \frac{b}{c} = \sum_{i=0}^{\infty} \frac{1}{\left(1 + \left(\frac{b}{c}\right)\right)^i}\, P_{t-1-i}$$

market conditions, he will use the information available to him about demand and supply conditions in generating his expectations about relevant variables for decision purposes. So it may be hypothesized that farmers have no different prices expectations from futures speculators, nor do those farmers who make no futures transactions have expectations different from those who do. If the price expectations of those not participating in the futures markets differ from the futures price, there is great incentive for them to enter. Thus, those out of the market likely have expected prices equal to the futures market price.

The rational expectations hypothesis also allows a solution for the second issue associated with using futures prices. The essence of the farmer's decision problem is to forecast the price for time t (the harvest price), given the information available through t-k (the time which the planting decision must be made). From a theoretical view-point, we can state that the futures price quote for harvest time in the preplanting period will provide available information and allow producers to observe a future (harvest) price at planting time. Peck (1976, p. 407) noted that,

> ". . . futures markets, with simultaneous trading in successive maturities, provide forward price(s) that could be used by a producer in formulating his production decisions, . . . one contract, the first in the new crop year, is of particular concern. Generally, this new crop futures is traded well in advance of the time that the production decision must be made. . . Unlike a price forecast, this is a forward price that is a market price. . . the producer could sell his expected output at that price."

Thus, we may incorporate equation (3.25) into system (3.28), since at planting time, a new crop futures price represents an observable realization of farmer's expectations. This is consistent with rational expectations[1].

Expectations are unbiased and producers are assumed to use futures prices in their production decisions. The decision is thus endogenous to the system and dependent upon futures prices. By including futures price as decision variables, we can set up a framework for empirically testing the hypothesis that producers have responded to futures prices in making their production decisions. The model also allows for partial adjustment to capture the technical constraints, institutional rigidities and persistence of farmer's habits involved in switching from one crop to the other. That is,

$$A_t^e = a + b\, P_t^e + \sum_{i=1}^{n} c_i Z_{it} + U_t$$

$$P_t^e = FP_{t-k}^{t} \tag{29}$$

$$A_t^e = A_{t-1}^{t} + r\,(A_t - A_{t-1}) \qquad 0 < r \leq 1$$

where all variables are as defined previously (page 14). Solving this system (3.29) gives

$$A_t = ar + br\, FP_{t-k}^{t} + (1-r)\, A_{t-1} + r \sum_{i=1}^{n} c_i Z_{it} + rU_t \tag{30}$$

which is to be statistically estimated and tested.

The appealing feature of using futures prices in supply response analysis is that they are "forward looking." However, prices generated by futures markets provide not only a forward pricing function but also the function of inventory (storage)

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[1]The rational expectations hypothesis (Muth's extended version) asserts that ". . . expectations of individual market participants (the subjective probability distribution of price outcomes) tend to be distributed for the same information set about the predictions of the theory (the objective probability distribution of price outcome)."

allocation. In the next section, the theory of storage behavior leading to the simultaneous equilibrium in the cash, futures and storage markets will be discussed.

II. THEORY OF SIMULTANEOUS EQUILIBRIUM IN THE CASH, FUTURE AND STORAGE MARKETS

A theory of simultaneous determination of cash and futures prices and consumption and inventoried quantities can be derived from the theory of hedging and speculation and theory of the price of storage. The development of the theory of hedging and speculation began with the view that the primary function of futures markets was the transfer of price risk from producers, merchants, processors and other inventory holders to speculators (Keynes, 1931 and Hicks, 1939). Speculators will buy (sell) contracts for future delivery only when the futures price (FP) is greater (less) than the expected cash price E(P) by an amount equal to or greater than the risk premium (R), Algebraically.

$$FP - E(P) \gtrless R \qquad (31)$$

Working (1948, 1949) argued that the Keynes-Hicks theory of hedging does not adequately explain a hedgers' actions. He asserted that hedging is not always insurance, whereby risk is shifted for a premium, but that hedging is really used when there is an expectation of a change in the cash-futures price relationship. Hedging is typically undertaken to exploit changes in the cash-futures price difference, rather than to simply ensure protection against an overall price change. Since hedgers take positions in both cash and futures markets, a knowledge of the basis is necessary in order to translate a given futures price into a probable price for cash delivery.

To illustrate a transaction in both cash and futures markets, assume that a hedge is carried from t_1 to t_2. The cash price and futures price at t_1 and t_2 are respectively CP_1, CP_2 and FP_1, FP_2. The total profit (loss) on the hedged position is

$$[\text{Profit (loss) in futures}] - [\text{loss (profit) in cash}]$$

$$= (FP_2 - FP_1) - (CP_2 - CP_1)$$

$$= (FP_2 - CP_2) - (FP_1 - CP_1)$$

$$= B_2 - B_1$$

$$= \triangle B \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \qquad (32)$$

where B_1, B_2 are the basis at time t_1 and t_2. $\triangle B$ is the change in the basis. The basis is the focal point for recognizing profitable opportunities.

The profit motive causes the futures market to reflect the cash market. The cash and futures prices tend to move up and down together, but as seen above they do not display equal fluctuations. Unequal fluctuations are caused by various market conditions including:

a) The overall supply and demand of a commodities;
b) the overall supply and demand of substitute commodities;
c) storage space available;
d) the behaviors of hedgers and speculators;
e) quality and transportation problems;
f) risk and uncertainty in expectations.

Hence, a change in the basis ($\triangle B$) can be positive, negative or zero. Although traders still face basis risk, it is less than price risk which is why traders watch basis behaviors and prefer basis to price risk.

The difference between the current cash price and the futures price reflects factors affecting the supply and demand for storage of these commodities. The narrowing basis is a reflection of the decreasing cost of storage as the delivery month approaches. Working (1949) argued that less (more) inventory is carried if the market shows a forward premium (discount) in relation to a futures delivery price. This premium is not a prediction that price will rise (fall), but rather a market-determined storage price. In his theory of the price of storage, the basis is used as a directly observable current price of storage[1].

Working's studies (1948 and 1949) on the theory of the price-of-storage emphasized the essential interdependence of the cash and futures markets. The theory of the price-of-storage views the basis ($FP_t^{t+k} - P_t$), whether positive or negative, as the market determined storage price. Further, it is postulated to be a direct function of current inventory levels. Working hypothesized that the supply-of-storage curve

$$FP_t^{t+k} - P_t = f(I_t) \qquad (33)$$

would have the shape depicted by ST_s in Figure 5.

Working's model allows for:

(i) Storage which may have a convenience yield (treated as a negative price of storage); and

(ii) Rising marginal costs of storage as the inventory levels increase or as available storage facilities become limited.

The essence of the price-of-storage (supply-of-storage) theory gives the direct connection between the cash and futures

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[1]As Working (1949) stated, ". . . the origin and prevalence of the term "carrying charge" in trade usage reflects the designated price differences as in fact equivalent to the price for "carrying" the commodity, or what may be called for economic analysis a price of storage."

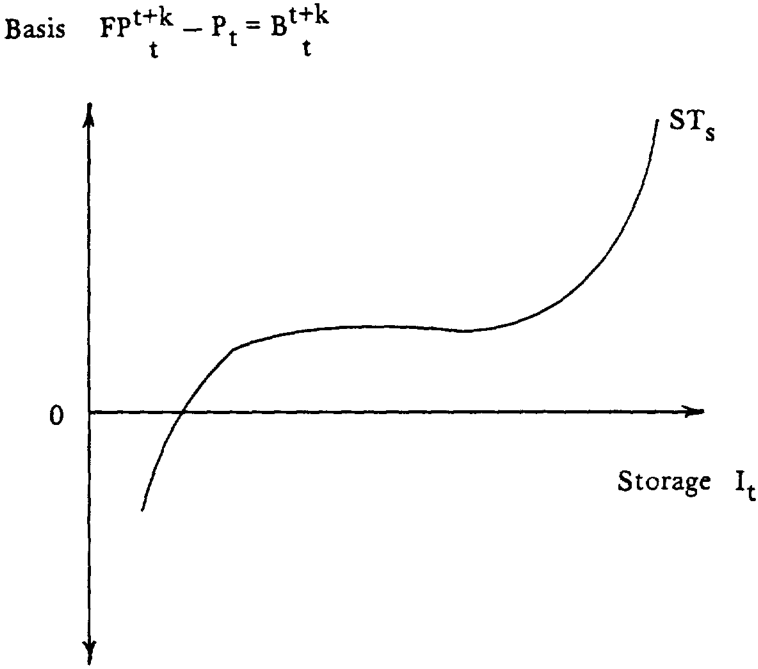


Figure 5. The supply-of-storage curve

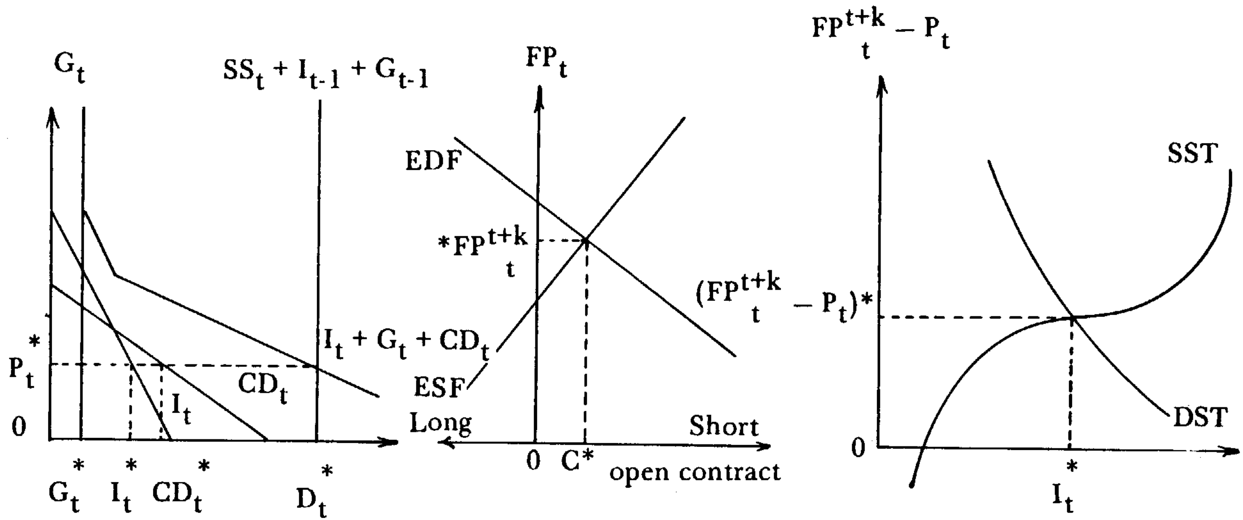
markets. As Working (1962, p. 455) stated,

> ". . . the main reason for listing the price of storage concept as making a significant step in the advance of economic science is that it seems capable of displacing the belief that spot prices are commonly little affected by changing distant expectations. That opinion is not founded on any factual observation, but has its "basis" in an assumption embodied in that conventional exposition of price formulation, an exposition that is ordinarily taken to mean that "supply" as a determination of spot price, means currently available physical supply."

Clearly, the usefulness of the theory of the price-of-storage is that it provides a direct explanation of intertemporal price relationships, and it can serve as a basis for the hypothesis of the interdependence of the three markets: cash market, futures market, and storage market.

To fully understand the role of the futures market in cash and storage price determination, the futures price must be considered as an endogenous variable whose value is determined simultaneously with cash and storage prices. For seasonally produced commodities, the average cash price for the year depends upon the demand and supply for the commodity. Futures prices, however, are determined directly by hedger and speculator demand for and supply of futures contracts. Market equilibrium as represented by the futures price, is the balance of net hedging and net speculation. The greater concern is that the futures price for the commodity and contract period is dependent upon the perceived demand and supply situation in the commodity market.

The spread between the cash and futures prices was defined above to be the price of storage. Theoretically, the equilibrium level of the price of storage ($FP_t^{t+k} - P_t$) and the size of the inventory (I_t) at a point in time are jointly determined by the supply of and demand for storage (see Figure 6). The demand for storage is related to consumption demand and the demand for inventory at the end of the period (Brennan, 1958, p. 52). The quantity of storage demanded is inversely related to the



a. Cash market equilibrium b. Futures market equilibrium c. Storage market equilibrium

Figure 6. Theoretical general equilibrium in cash, futures and storage markets

price of storage. The price of storage influences the amount of storage over the season. If the difference between futures and cash price exceeds the cost of storage, this will provide an incentive to store the commodity by selling futures contracts and holding the commodity for future delivery. A negative price of storage, referred to as an inverted market situation, is usually considered to reflect a current shortage of the commodity.

Within this framework, the equilibrium situation is presented diagrammatically in Figure 6. In Figure 3.6a, the average cash price for the year is dependent on the total supply (SS_t + I_{t-1} + G_{t-1}) and total demand ($CD_t + I_t + G_t$). P_t^* is the equilibrium price and I_t^*, G_t^*, CD_t^* are equilibrium quantities of private inventory, government stock and current disappearance, respectively.

In the futures market, traders can be classified into two groups: hedgers and speculators. In this discussion, hedgers are described as traders who have taken the opposite position in the cash and futures markets. Speculators are traders who have both bought and sold futures at different times. Both hedgers and speculators can be in "long" or "short" or "both" positions.[1]

Futures market equilibrium is achieved when demand for and supply of futures contracts are equal, i.e.,

$$H_L + S_L = H_S + S_S \qquad (34)$$

where:

H_L = net long hedging

H_S = net short hedging

S_L = net long speculation

S_S = net short speculation

[1]Buying futures contracts is "long" while selling futures is "short".

Rewrite equation (3.34) as

$$S_L - S_S = H_S - R_L \qquad \text{or} \tag{35}$$

$$EDF = ESF \tag{36}$$

where:

EDF = excess demand for futures contracts for speculators

ESF = excess supply of futures contracts by hedgers

In Figure 6b, the intersection of the EDF curve with the ESF curve gives the equilibrium level of the futures price $*FP_t^{t+k}$, and the net commitment C* in terms of open contracts measured in units of the cash commodity in that period.

The storage market equilibrium, is shown in Figure 6c. Efficiency in storage occurs when supplies are distributed through time so that the marginal return to storage in each period is equal to the marginal cost of bringing a commodity to that period.

Simultaneous equilibrium occurs when there exists a proper price-quantity linkage between the three markets. In general, if all three markets are in competitive equilibrium, then in Figures 6a, 6b, and 6c, the difference between the equilibrium cash price, (P_t^*) and the futures price, ($*FP_t^{t+k}$) is equal to (FP_t^{t+k} – P_t)*. The equilibrium level of inventory, I_t^*, in Figure 6a must be equal to the equilibrium level of inventory, I_t^* in Figure 6c. Since the basis will affect the level of inventory and the economic return to storage, the simultaneous determination of cash and futures prices reflects the interdependence of the three markets.

The theory developed is particularly relevant to those commodities produced once but stored and utilized throughout the year.[1]

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[1]The model is restricted to the interharvest period of a periodically produced storable commodity. The model could be made more general by adding a supply equation to incorporate harvest changes. This will be considered in a later section.

The behavior of hedgers, merchants and speculators affects cash and storage markets. The prices generated from cash and futures markets further give market-determined prices of storage, which provide incentives or determine stocks to be stored. In essence,

a. Hedgers carry stock and reduce their risk by selling futures in an amount equal to the quantity of stock they intend to carry over to the next period. Hedger's actions are determined by cash and futures prices.

b. Merchants carry unhedged stocks and react to the present cash price and the expected cash price in the futures. They are really speculators in cash.

c. Speculators buy futures contracts according to the level of the futures price relative to the expected cash price. They do not carry stocks of the commodity.

It is assumed that markets are perfectly competitive. Expectations about the spot price for future periods are given. The model has three markets distinguished as follows:

1) The Storage Market

The demand for storage arises from those who plan to be holders of stocks in the next period. This includes merchants who hold unhedged stocks and speculators who plan to take delivery on futures contracts.

The supply of storage is provided by those who hold stock during the present period, including hedgers who hold hedged stocks and merchants who hold unhedged stocks.

2) The Futures Market

The supply of futures contracts is provided by short hedgers and speculators. The demand comes from long hedgers and speculators in futures contracts.

3) The Cash Market

Demand arises from present consumption while supply in the spot market must be allocated to current consumption and storage.

The market interdependence relationships and the three-market equilibrium can be presented diagrammatically using Figure 7. In the diagram, consumption demand DD is a decreasing function of the spot price. It is not affected by a change in the futures price. R_1R_1 is the supply of (and demand for) unhedged storage by merchants. It is drawn for a given cash price P_1 and with a higher cash price it would shift to the left. HS is the supply of hedged stock by hedgers and is drawn for a given cash price P_1. For a higher cash price, it would also shift to the left. DF is the demand for futures contracts by speculators. The total demand for storage by speculators and merchants (DST) is the horizontal summation of R_1R_1 and DF at each future price level. The total supply of storage by hedgers and merchants (SST) is the horizontal summation of R_1R_1 and HS.

The equilibrium futures price FP_1^* is determined by the intersection of DF and HS, i.e., the equality of demand for and supply of futures contracts, or alternatively of DST and SST. The total quantity of storage forthcoming at FP_1^* is 00_2, of which 00_1 is supplied by hedgers and $0_1\ 0_2$ is supplied by merchants. Since total stock of the commodity to be allocated is OH, this leaves O_2H for present consumption. Given the consumption demand curve DD, the cash price is P_1^*, and the equilibrium price of storage is B_1^* which is equal to $FP_1^* - P_1^*$. This price difference will also affect storage behaviors and hence other components in the system since these markets dynamically and simultaneously interact.

There are three equilibrium conditions in this model. Total stocks must be allocated between storage and present consumption demand and the cash and futures prices must equate

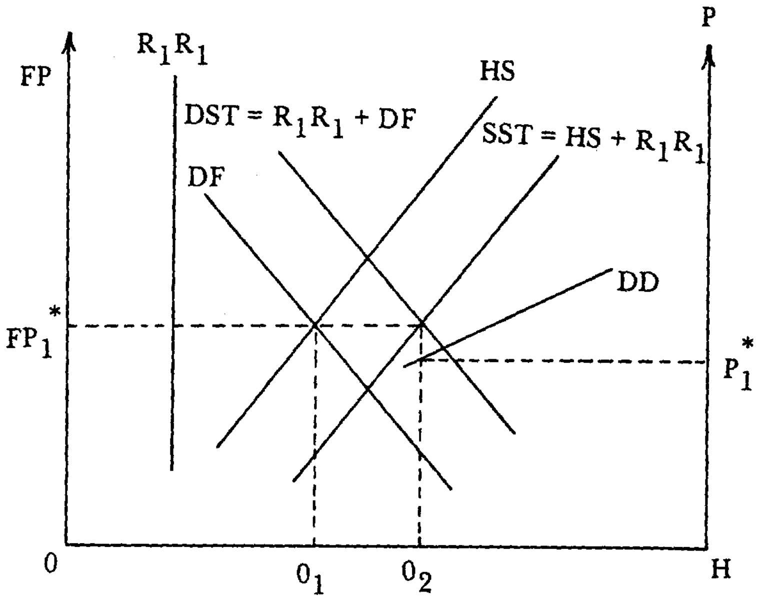

Figure 7. Determination of cash and futures prices with storage market interdependence[a]

[a]The quantities of storage and of futures contracts are measured from left to right on the horizontal axis and consumption is measured in the opposite direction. The futures price is shown on the left-hand ordinate and the cash price is shown on the right-hand side.

demand and supply in the cash and futures markets respectively For empirical purposes, the role of futures price can be taken into account in the storage market since the futures-cash price differential reflects price of storage. Therefore, the simultaneous equilibrium in the cash and storage markets also captures the interdependence of the cash, futures and storage markets.

For a general equilibrium to exist, there must be a set of prices in these three markets allowing all these markets to clear. Under such conditions the consumers and the firms maximize their utility. In the context of risk management, the simultaneous equilibrium of the three markets is indicative of the process of finding an equilibrium point between risk-bearers and risk-averters.

III. CONCLUSIONS – GENERAL MODEL SPECIFICATION

Along with the theoretical background and the review of previous modeling efforts, a model which takes into account the role of futures price and captures the simultaneous equilibrium in the cash and storage markets for corn and soybeans can be constructed and represented in mathematical form. In the cash market, the supply equations need to contain both the prices of corn and soybeans. Demand side equations, need to contain not only the commodity prices but also the price of livestock since both corn and soybeans are used as feeds.

The interdependence of cash, futures and storage markets can be captured by treating the futures price as endogenous in the demand and supply of storage equations (since the difference between cash and futures price refers to price of storage).

The mathematical model which is consistent with the conceptual framework of Figure 7 for the interrelated corn and soybean sectors can be represented as

corn supply $Q_c^s = f_1 [P_c, P_s, \underset{\sim}{G}, \underset{\sim}{X}] + \varepsilon_1$ (37)

corn demand $Q_c^d = f_2 [P_c, P_{ls}, \underset{\sim}{X}] + \varepsilon_2$ (38)

soybean supply $Q_s^s = f_3 [P_c, P_s, \underset{\sim}{G}, \underset{\sim}{X}] + \varepsilon_3$ (39)

soybean demand $Q_s^d = f_4 [P_s, P_{ls}, \underset{\sim}{X}] + \varepsilon_4$ (40)

supply of corn storage $ST_c^s = f_5 [FP_c - P_c, FP_s - P_s, \underset{\sim}{G}, \underset{\sim}{X}] + \varepsilon_5$ (41)

Demand for corn storage $ST_c^d = f_6 [FP_c - P_c, \underset{\sim}{G}, \underset{\sim}{X}] + \varepsilon_6$ (42)

Supply of soybean $ST_s^s = f_7\,[FP_s - P_s, FP_c - P_c, G, X] + \varepsilon_7$ (43)

storage

Demand for soybean $ST_s^d = f_8\,[FP_s - P_s, G, X] + \varepsilon_8$ (44)
storage

Basis identities (price of storage)

$$B_c = FP_c - P_c \quad (45)$$

$$B_s = FP_s - P_s \quad (46)$$

Storage market equilibrium

$$ST_c^s = ST_c^d \quad (47)$$

$$ST_s^s = ST_s^d \quad (48)$$

Market clearing Identities

$$Q_c^d + ST_c^d = Q_c^s \quad (49)$$

$$Q_s^d + ST_s^d = Q_s^s \quad (50)$$

where

Q_c^s = quantity supplied of corn

Q_c^d = quantity demanded for corn

Q_s^s = quantity supplied of soybeans

Q_s^d = quantity demanded for soybeans

ST_c^s = quantity supplied of storage for corn

ST_c^d = quantity demanded for storage for corn

ST_s^s = quantity supplied of storage for soybeans

ST_s^d = quantity demanded for storage for soybeans

P_c = cash price of corn

P_s = cash price of soybeans

FP_c = futures price of soybeans

FP_s = futures price of corn

B_c = corn basis (price of storage)

B_s = soybean basis (price of storage)

P_{ls} = price of livestock and its products

$\underset{\sim}{G}$ = government policy variables

$\underset{\sim}{X}$ = other exogeneous variables

The variables endogenous to this model are:

$$Q_c^s, Q_c^d, Q_s^s, Q_s^d, ST_c^s, ST_c^d, ST_s^s, ST_s^d, P_c, P_s, FP_c, FP_s, B_c, B_s.$$

The model specified above disregards the temporal distribution of adjustments. However, due to the inherent time lag in agricultural production, the model should be formulated under the assumption that the current price will not have a significant influence on current production. Instead, producers base their production decisions on expected price. Several hypotheses can be made that take into account formation of price expectations.

If the distributed lag hypothesis is used, the supply equations (37), (39) use lagged prices as expected prices. However, within the spirit of the rational expectations hypothesis, in this research, futures prices will be utilized as expected prices in making production decisions for corn and soybean producers.

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