Gap ratio effect on flow characteristics behind side-by-side cylinders of diameter ratio two

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Abstract

This study investigates the vortex interaction, the spatial distributions in the flow and the streamwise evolution of the spectral amplitude along the shear layer behind side-by-side cylinders of diameter ratio two at different gap ratios via dye flow visualization and particle image velocimetry (PIV). The frequency responses are measured at Re = 1000, 2000, 5000 as the gap ratio changes. Velocity measurements are made and analyzed at Re = 1000 for gap ratios of 1.25, 0.75 and 0.25. It is found that, as the gap ratio increases, the Strouhal number of the narrow wake decreases monotonously but that of the wide wake increases also in the monotonous way. The gap flow is always stably deflected toward the small cylinder. For side-by-side cylinders of diameter ratio two and Re = 1000, two different vortex interaction scenarios are found leading to two different flow categories. The critical gap ratio for diameter ratio two is slightly smaller than that for equal diameter. The frequency ratio of the narrow and wide wakes depends strongly upon the gap ratio and the diameter ratio; but is independent of the Reynolds number studied. This frequency ratio is related to the ratio of the averaged streamwise distance of the local maxima of each spectral component. For side-by-side cylinders of diameter ratio two, two flow characteristics are modified relative to that of a single cylinder. First, the onset locations of the shear layer instability. Second, the spatial growing rates of the shear layers. For the side-by-side cylinders of diameter two (D/d = 2), the influence on the wide wake is more significant but is less pronounced on the narrow wake. Besides, the influences on both the narrow and the wide wakes are even pronounced for D/d = 2 than those for D/d = 1.

1. Introductions

Unsteady flows passing through the cylinder couple of various arrangements show diversified fluid phenomena within a wide range of Reynolds numbers including the shedding of vortices, the mutual interactions and the loads on these cylinders. The related flow phenomena attracted many researchers because of the fundamental importance in the industrial applications [26,27]. For side-by-side cylinders of equal diameter, the types of vortex interaction and evolution are strong functions of the gap ratio and Reynolds number [2,14,20,23,24]. Such interactions may lead to different vortex frequencies, flow structures far downstream and dramatic change of the loads acting on each cylinder [1,11].

Behind the side-by-side cylinders of equal diameter, the basic flow structures are categorized as follows. First, only one single vortex street exists if they are spaced very closely (G < 0.3). In this case, the shedding frequency is about half of that behind a single cylinder at the same Reynolds number. Secondly, a vortex street behind each cylinder is observed while the gap ratio (G) is equal to or greater than 2.5. These two vortex streets interact very weakly and may shed in phase or out of phase at random time intervals [1,13,23]. Third, while two cylinders are spaced with intermediate gap ratios, the flow structures depend upon the Reynolds numbers, the gap ratio and the experimental conditions [3]. For the gap ratio 0.3 < G < 2.0, formation of a wide and a narrow wake behind each cylinder is observed [3,24]. The well known flow pattern is the stably biased gap flow. Under some situations, the gap flow may switch upwards or downwards at intermittent and random time intervals [1,8,11,17]. The switching timescale is several orders of magnitude longer than those of the vortex shedding and the shear layer instability [10]. From the practical viewpoints, the third category receives more attention because the loadings (mean, fluctuating drag and lift) on the cylinders upstream of the narrow and the wide wakes exhibit appreciable differences.

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For the cylinder couple of diameter ratio two, Lam et al. [15] and Ko et al. [12] measured in a wind tunnel the mean base pressure coefficients, the velocity spectra, the energy distributions in the wake region at $G' = 0.75$, $L' = 0.0$ and $G' = 0.4$, $L' = 0.25$, respectively, for $Re = 5.0 \times 10^4$ based on the diameter of the large cylinder. Lam et al. [15] revealed the detailed interaction mechanism between the vortices of the deflected gap flow and those along the free stream sides by hot wire through the conditional average technique. Ko et al. [12] pointed out that the interactions between the gap vortices with outer vortices played an important role in the downstream development of the wakes. Both results showed that the interactions between gap vortices are more complicated than those for cylinders of equal diameter. Recently, Gao et al. [4] investigated the wake structure behind two side-by-side circular cylinders of diameter ratio 1.5 at three intermediate gap ratios by PIV technique. They focused on elucidating the asymmetrical mean flow characteristics for $Re = 1200$, 2400 and 4800. They found the changeover of the gap flow at $G' = 0.45$ and $Re = 1200$ and provide a plausible interpretation, which is similar to that of Brun et al. [3], to explain the generation mechanism of the biased gap flow. To date, very little study has been focused on this subject probably due to the relatively complicated interacting flow structures or insufficient analyzing techniques.

In the present study, the Reynolds number 1000 is selected because the vortex formation length behind the large cylinder is the longest and the wake behind is more likely to be affected by the proximity of a small cylinder. The vortex formation length is defined as the streamwise distance between the cylinder center and the location where the spectral amplitude of velocity fluctuation reaches the maximum. In our preliminary studies for $D/d > 4$ arranged side-by-side, the small cylinder merely provides local disturbance and minor modification on the neighboring separating shear layer of the large cylinder. For the case of diameter ratio two, the small cylinder is expected to affect the separating shear layers of the large cylinder and vice versa, leading to different flow categories far downstream. So far, the vortex interaction and the influencing mechanisms are still relatively unexplored and require further investigations. Thus, in this study, the flow structures behind the side-by-side cylinders of diameter ratio two $(D/d = 2)$ spaced at three gap ratios $(G' = 0.25, 0.75$ and $1.25)$ are studied by the PIV system. The frequencies are measured on the free stream sides along the shear layer of both cylinders at $Re = 1000$, $2000$ and $5000$ to show the variation trends of the Strouhal numbers as the gap ratio changes. Velocity measurements are made and processed for $Re = 1000$ to demonstrate the spatial distributions of the spectral amplitude over the flow domain for different gap ratios. Also, the spatial growth and the delay distance of the onset of the shear layer instability along the outer shear layers of both cylinders are analyzed to disclose the mechanism leading to different frequencies of the narrow and the wide wakes.

2. Experimental system and techniques

2.1. Coordinate system and flow parameters

Fig. 1 defines the flow parameters and the coordinate system of the side-by-side cylinders of diameter ratio two placed in a uniform stream of magnitude $U$. For the two-cylinder system, the origin is located at the center of small cylinder with diameter $d$ ($d = 1$ cm). The diameter of large cylinder is denoted as $D$. The characteristic frequencies of the narrow and wide wakes are defined by $f_n$ and $f_w$, respectively. In this study, the diameter $D$ and the vortex shedding frequency $f_D$ of a single cylinder with diameter $D$ are employed as the reference length and frequency. The positive $X$ and $Y$ directions are defined along the inflow direction and upward normal to the incoming flow, respectively. The plane at $Z = 0$
Then, the unsteady flow structures around the cylinders are investigated in the stagnation point of each cylinder. The Rhodamine G6 fluorescent dye is released naturally from these holes as the fluid traces. The side walls, with beveled leading edges, pin-support the side-by-side cylinders at their centers from both ends. Before installing the cylinders, all the velocity spectra, measured in free stream at various locations along the spanwise and transverse directions, show only wide band characteristics at noise level without any spectral peak. The streamwise turbulence intensity measured across the shear layer. In the present study, the maximum mean velocity gradient at X/D = 1.0, the displacement bias error, estimated from Westerweel [21] for the of simple linear shear flows, results in an error of 0.96% U in the velocity measurements of the flow field are implemented. The averaged size of particle image is around 1.25% of the interrogation window size. Based on the particle image size and the interpolation kernel and the algorithm for evaluating the velocity vectors is based on the cross correlation and optimized with a 50% overlap of the interrogation window. Within the flow domain, there are 4260 velocity vectors/frame are detected to be valid out of 4590 uniform grid points. To elucidate the detailed flow information, the velocity signals within the flow field are extracted from the sequential images obtained from PIV system. The velocity data measured by the PIV system are reliable for all the cases studied herein. Within the flow domain, the hollow glass beads having a diameter of 5–8 μm with a specific weight of 1.03 are used as seeding particles. Each acquisition duration contains 4000 frames which includes 15–20 Tw and 75 Tw, respectively. The physical field of view is 5D by 5D. Here D is the diameter of large cylinder. The size of interrogation window is 16 by 16 pixels and set to be adaptive so that the bias of measured velocity due to large velocity gradients (across the shear layer) can be reduced. The side walls, with beveled leading edges, pin-support the side-by-side cylinders at their centers from both ends. Before installing the cylinders, all the velocity spectra, measured in free stream at various locations along the spanwise and transverse directions, show only wide band characteristics at noise level without any spectral peak. The streamwise turbulence intensity measured across the shear layer. In the present study, the maximum mean velocity gradient at X/D = 1.0, the displacement bias error, estimated from Westerweel [21] for the of simple linear shear flows, results in an error of 0.96% U in the velocity measurements of the flow field are implemented. The averaged size of particle image is around 1.25% of the interrogation window size. Based on the particle image size and the interpolation kernel and the algorithm for evaluating the velocity vectors is based on the cross correlation and optimized with a 50% overlap of the interrogation window. Within the flow domain, there are 4260 velocity vectors/frame are detected to be valid out of 4590 uniform grid points. To elucidate the detailed flow information, the velocity signals within the flow field are extracted from the sequential images obtained from PIV system. The velocity data measured by the PIV system are reliable for all the cases studied herein. Within the flow domain, there are 4260 velocity vectors/frame are detected to be valid out of 4590 uniform grid points. To elucidate the detailed flow information, the velocity signals within the flow field are extracted from the sequential images obtained from PIV system. The velocity data measured by the PIV system are reliable for all the cases studied herein. Within the flow domain, there are 4260 velocity vectors/frame are detected to be valid out of 4590 uniform grid points. To elucidate the detailed flow information, the velocity signals within the flow field are extracted from the sequential images obtained from PIV system. The velocity data measured by the PIV system are reliable for all the cases studied herein. Within the flow domain, there are 4260 velocity vectors/frame are detected to be valid out of 4590 uniform grid points.

![Fig. 1. Experimental model set-up and comparisons of the measured data with theory.](image)

### 2.2. Model arrangements, measurement and visualization techniques

The experiments were performed in a recirculating water channel with a cross section of 0.45 m (spanwise) × 0.4 m (streamwise). The aspect ratios W/D and the blockage ratio of both cylinders are 22.5%, 45% and 7.5%, respectively. The side walls, with beveled leading edges, pin-support the side-by-side cylinders at their centers from both ends. Before installing the cylinders, all the velocity spectra, measured in free stream at various locations along the spanwise and transverse directions, show only wide band characteristics at noise level without any spectral peak. The streamwise turbulence intensity measured in the free stream is about 0.78%. The boundary layer thickness at the cylinder locations is measured to be 3.14% W. Along both the spanwise and the transverse directions, the spatial uniformity of free stream velocity is measured to be 97.2%, excluding the boundary layer of the side walls and the bottom wall. Dye visualization of the unsteady flow structures behind side-by-side cylinders is conducted by the laser sheet technique. The surface of each cylinder, tiny holes of 0.5 mm diameter on the mid-span plane (Z = 0) are oriented at ±60° from the forward stagnation point of each cylinder. The Rhodamine G6 fluorescent dye is released naturally from these holes as the fluid traces. Then, the unsteady flow structures around the cylinders are recorded continuously by a CCD camera (Sony DCR-PC100) with a framing interval of 1/30 s.

A high-speed PIV system integrated by the ProVISION software is employed to measure the instantaneous flow structure. The hardware of this PIV system contains an infrared pulse laser (XS-IR4), a high-speed CMOS camera (X-Stream XS4, manufactured by Redlake) and an X-Stream Timing hub. The maximum camera speed is 5000 frames/s (fps) at full resolution (512 by 512 pixels). Each acquisition duration contains 4000 frames which includes 15–20 Tw and 75 Tw, respectively. The physical field of view is 5D by 5D. Here D is the diameter of large cylinder. The size of interrogation window is 16 by 16 pixels and set to be adaptive so that the bias of measured velocity due to large velocity gradients (across the shear layer) can be reduced. Analysis of the velocity vectors is based on the cross correlation and optimized with a 50% overlap of the interrogation window. Based on the above settings, the maximum displacement of two consecutive particle images is less than one quarter of the interrogation size [9]. Details of the interpolation kernel and the algorithm for evaluating the velocity vectors can be found in the user manual of ProVISION-XS [7]. The averaged size of particle image is around 1.25% of the interrogation window size. Based on the particle image size and maximum mean velocity gradient at X/D = 1.0, the displacement bias error, estimated from Westerweel [21] for the of simple linear shear flows, results in an error of 0.96% U in the velocity measurement across the shear layer. In the present study, the maximum velocity gradient occurs at in the near wake. In Fig. 1(b), mean streamwise velocity profile of the upper separating shear layer behind the large cylinder is measured at X/D = 1.0 and compared with the theoretical hyperbolic tangent profile [18]. For the definitions of U∗ and y∗ in Fig. 1(b), please refer to the Nomenclature section. Coincidence of the measured data with the theoretical curve across the shear layer further ensures that the velocity data measured by the PIV system are reliable for all the cases studied herein. Within the flow domain, there are 4260 velocity vectors/frame are detected to be valid out of 4590 uniform grid points.

To elucidate the detailed flow information, the velocity signals obtained from PIV system. The wake flows studied, the frequency of interest ranges between 0.3 Hz and 3.0 Hz; thus the 100 Hz framing rate of the PIV system is free from aliasing error. The frequency resolution is 0.025 Hz (Δf) within a bandwidth of ±0.5 Hz. Under these sampling conditions, the temporal and the frequency resolutions are sufficient enough to resolve the unsteady flow characteristics.

Three sets of the sampled data are averaged to acquire the statistically time-averaged flow information. At each grid point over the flow domain, the velocity fluctuations are acquired by subtracting the time-averaged velocity from the instantaneous velocity. The power spectral density (PSD) functions of the velocity fluctuations are evaluated by fast Fourier transform (FFT) with a hamming window of which the size equals the sampling period. According to Eq. (1), the spectral amplitude equals the square root of the area underneath the ensemble-averaged PSD about the central frequency (f) within a bandwidth of ±Δf.

\[ \tilde{u}(f) = \sqrt{\frac{1}{2\Delta f} \int_{f - \Delta f}^{f + \Delta f} |PSD(f)| df} \]  

At each grid point over the entire flow domain, an average of three PSD yields the ensemble-averaged spectral amplitudes at each dominant frequency.
3. Results and discussions

3.1. Frequency responses and flow structures for D/d = 1.0

Fig. 2(a) demonstrates the relation between the Strouhal numbers ($St_n$ and $St_w$) and the gap ratio for side-by-side cylinders of equal diameter. In Fig. 2(b) and (c), the point Q is near the location where the vortices A and B are formed and the point P is at downstream location where the shear layers roll up into a vortex D or cluster of vortices A, B and C. Depending on the flow structures at the gap ratios and Reynolds numbers studied, the streamwise locations of the points P and Q may vary; but their elevations are along the free stream sides of both shear layers to have good signal quality and correctly demonstrate the characteristic frequencies of the wide and the narrow wakes.

Within $0.5 \leq G^* \leq 1.5$ in Fig. 2(a), the Strouhal number ($St_n$) of the narrow wake decreases monotonously with increasing gap ratio while that ($St_w$) of the wide wake increases monotonously. Within this range ($0.5 \leq G^* \leq 1.5$), the values of $St_n$ and $St_w$ deviate largely from each other, corresponding to a narrow and a wide wakes behind the cylinder couple. Typical flow pattern is given in Fig. 2(b). While the gap ratio $G^*$ is greater than 1.5, both Strouhal numbers reach the same value ($St_n = 0.211$), representing two nearly independent vortex streets shed behind each cylinder. At $G^* = 0.25$ and $Re = 1000$, the flow pattern is depicted in Fig. 2(c). In Fig. 2(a) at $G^* = 0.25$, the small Strouhal number ($St_n$) of 0.11 (measured at P and denoted by $\triangle$) represents the characteristic frequency of the wide wake; while the large Strouhal number ($St_w$) of 0.42 (measured at Q and indicated by $\triangledown$) denotes the characteristic frequency of the narrow wake. In Fig. 2(c), the narrow wake merges with the stably deflected gap flow and the frequency corresponding to $St_n$ is not detectable in the far wake region at $G^* = 0.25$. In other words, at $G^* = 0.25$, only a single vortex street exists in the far wake region. This flow structure is similar to that demonstrated by Xu et al. [24] at $G^* = 0.3$ and $Re = 350$ as well as those reported in previous literature for $G^* \leq 0.3$ within a wide range of Reynolds number [3,11]. Within the gap ratio $0.5 \leq G^* \leq 5.0$, the variation trend of Strouhal numbers in Fig. 2(a) and the classification of flow structures are also similar to those reported in Kiya et al. [11] and Kim and Durbin [10] at different Reynolds numbers.
3.2. Frequency responses and flow structures for $D/d = 2.0$

Fig. 3(a) illustrates the Strouhal numbers of the wakes behind the cylinder couple of diameter ratio two at $Re = 1000$, $2000$, $5000$ while the gap ratio ranges from $0.5$ to $5.0$. For $G^* \geq 1.25$, typical flow structure is shown in Fig. 3(b) and the $St_n$ and $St_w$ denote the frequencies of the narrow and the wide wake measured at $Q$ and $P$, respectively. In Fig. 3(a), as the gap ratio increases, the values of $St_n$ decrease monotonously to the dashed line level around $G^* = 5.0$. Within $0.5 \leq G^* \leq 1.25$, the gap flow is deflected stably toward the small cylinder and a typical flow structure with is given in Fig. 3(c) for $G^* = 0.75$. From Fig. 3(b)–(d), it is clear that the difference between $St_n$ and $St_w$ is related to the degree of mutual interactions between the narrow and the wide wakes for $0.5 \leq G^* \leq 1.25$. In Fig. 3(d) at $G^* = 0.25$, the stably deflected gap flow merges with the narrow wake shortly behind the small cylinder and only one single frequency is measured in the far wake region. This represents a single wide wake far downstream. Because of the diameter ratio two, $St_n$ is only one half of $St_w$ in Fig. 3(a) at $G^* = 0.25$ for all Reynolds numbers studied. At $G^* = 0.25$ and $D/d = 2.0$, the single wide wake flow structure is similar to that reported by Xu et al. [24] for $D/d = 1.0$. The results in Fig. 3(a) are also similar to that of Yokoi and Hirao [25] at $D/d = 2.0$ and $Re = 500$ in the towing tank experiments.

Compared with the data in Fig. 2(a) at $G^* < 1.5$, the rapid increase of $St_n$ and quick convergence to the value of $St_D$ in Fig. 3(a) clearly indicates that the influence on the wide wake is more significant for $D/d = 1$ than that for $D/d = 2$. Similarly, in Fig. 3(a), the slow decrease and mild convergence of $St_n$ to the value of $St_D$ demonstrates that the influence on the narrow wake is less pronounced for $D/d = 2$ than that for $D/d = 1$.

3.3. Frequency ratio of the narrow and wide wakes for $D/d = 2.0$

Fig. 4 illustrates the frequency ratio ($f_{nl}/f_{nw}$) as functions of the gap ratio at the Reynolds numbers studied for $D/d = 2.0$ (solid) and $1.0$ (open). In spite of the variation of Reynolds numbers, the frequency ratios in Fig. 4 are collapsed onto two independent curves respectively for $D/d = 1.0$ and $2.0$. Evidently, the frequency ratio $f_{nl}/f_{nw}$ is strong functions of the diameter ratio ($D/d$) and gap ratio ($G^*$) but is nearly independent of the Reynolds numbers studied herein.

Within $0.25 \leq G^* \leq 1.5$ for both $D/d = 2.0$ and $1.0$, the frequency ratio decreases dramatically and monotonously as the gap ratio increases. The slope for $D/d = 1.0$ is much steeper than that for $D/d = 2.0$. This also demonstrates that the influences on vortex shedding of the narrow and the wide wakes are more significant
for \( D/d = 1 \) than for \( D/d = 2 \). At large gap ratios (\( G^* > 2.0 \)), the frequency ratio reaches about 2.0 for \( D/d = 2 \) and 1.0 for \( D/d = 1 \), respectively. Though two curves reach two different values of \( f_n/f_w \), they both represent two independent vortex streets behind each cylinder. For side-by-side cylinders of diameter ratio two spaced at \( G^* = 0.75 \), Lam et al. [15] measured a frequency ratio of 2.4 at a much higher Reynolds number (5.0 \( \times 10^5 \)). At \( G^* = 0.75 \), the averaged data in Fig. 4 yields a frequency ratio about 3.0. The discrepancy in \( f_n/f_w \) may be caused by an order of magnitude difference in the Reynolds number.

3.4. Interacting flow structures for \( G^* = 1.25 \) and 0.75

Fig. 5 illustrates the typical flow structures at selected instants for \( G^* = 1.25 \). Note that \( r^* = t/T_w \) and \( r^* = 0 \) is arbitrarily chosen. In all the pictures, the symbols \( A \) and \( D \) denote the vortices generated from the free-stream sides of the large and the small cylinders, respectively; while \( B \) and \( C \) represent the gap vortices generated from the inner shear layers of both cylinders. The normalized period of the wide wake behind the large cylinder is 1.0 (Fig. 5(a)–(f)) and that of the small cylinder is around 0.363 (Fig. 5(a)–(c) and (b)–(d)), corresponding to \( f_{w0} = 1.0 \) and \( f_{w0} = 2.77 \), respectively. Here \( f_w = f_w/f_{w0}, f_n = f_n/f_{w0} \) and \( f_A \) is the shedding frequency of a single cylinder of diameter \( D \). In Fig. 5, the gap flow is observed to deflect stably toward the small cylinder. Along the upper shear layer of Fig. 5, small frills are formed intermittently around the edge of the vortex \( A \). Since the frills shown in Fig. 5 occur intermittently at \( Re = 1000 \), the spectral peak at Bloor–Gerrard instability frequency \( (f_{in}f_{f0} \times (Re)^r) \), where \( r = 0.5 \) or 0.87 [22] is difficult to detect along the upper shear layer. Note that the Bloor–Gerrard instability is first found by Bloor and Gerrard [5], the reader can be referred to this article for details. Velocity spectra taken along the upper shear layer in the near wake region over a range of streamwise locations show only a spectral peak centered at the frequency \( f_{w0} \) in Fig. 5(g). This coincides with the result of Unal and Rockwell [19] that the onset of Bloor–Gerrard instability frequency is not apparent for \( Re < 1900 \). In Fig. 5, along the upper shear layer, the frills are continuously engulfed into the vortex \( A \) which sheds downstream at a frequency \( f_{w0} \). No vortex pairing is observed along the upper shear layer at \( G^* = 1.25 \). Within the region \( X/D < 2.5 \), the gap vortices \( B \) and \( C \) form almost at the same phase and move nearly at the same speed. In Fig. 5(b)–(d), coalescence of the vortices \( B1 \) and \( B2 \) are clearly observed within 2.5 \( \times X/D < 3.0 \); subsequently, a vortex \( B3 \), moving at a higher speed, is observed to catch up with the preceding coalescent vortex \( (B1 + B2) \). Eventually, in Fig. 5(e) and (f), the vortices \( B1, B2 \) and \( B3 \) amalgamate into a large vortex \( B1 + B2B3 \) and shed alternately downstream at the same frequency as the vortex \( A \). Behind the small cylinder, the vortices \( C \) and \( D \) is observed to shed alternately downstream at a frequency \( f_{w0} \) (Fig. 5(a)–(c) and (b)–(d)).

For \( G^* = 0.75 \), the interaction scenarios of vortices \( A, B, C \) and \( D \) in Fig. 6 are similar to those described in Fig. 5. There exist a wide wake behind the large cylinder and a narrow wake behind the small cylinder whose normalized periods are 1.05 and 0.37, respectively. The main differences are three folds. First, in Fig. 6(a)–(c), along the upper shear layer, the small scale frills form regularly at a time interval about \( r^* = 0.37 \) = \( 1/f_{w0} \). The velocity spectrum of Fig. 6(d), measured near the upper shear layer, shows only two spectral peaks of which the frequency \( f_{w0} \) is about three times the frequency \( f_{w0} \). In Fig. 6(a)–(c), the low frequency (\( f_{w0} \) corresponds to the frequency of vortex \( A \); while the high frequency (\( f_{w0} \) represents that of the vortices \( C \) and \( D \). This clearly implies that the frills in form of small scale vortices are induced by the formation of the vortices \( C \) and \( D \) and are entrained continuously into the vortex \( A \). Second, the gap flow stably deflects more obviously toward the small cylinder. Third, the high frequency of the narrow wake is about three times the low frequency of the wide wake. In Figs. 5 and 6, the frequency ratios (\( f_n/f_{w0} = 2.77 \)) at \( G^* = 1.25 \) and \( (f_n)/ f_{w0} = 2.93 \) at \( G^* = 0.75 \) correspond to the coalescence of three B vortices into a large scale vortex shedding at the same frequency as the vortex \( A \). Note that, in the present study, \( f_{w0} = 1.0 \) for these two cases. Because of this interaction scenario, the wide wake will experience the one-third sub-harmonic lock-on while the large cylinder is perturbed within a frequency band around \( 3f_{w0} \) [16]. In Figs. 5 and 6, though the two-wake flow structures with two distinct frequencies are similar to that reported by Xu et al. [24] at \( G^* = 0.3 \) and \( Re = 1000 \) for side-by-side cylinders of equal diameter, they did not provide the interaction scenarios of the gap vortices and the coalescence of the vortices \( B \). In addition, the vortex interaction scenarios shown in Figs. 5 and 6 are different from that of Williamson [23] at \( Re = 200 \) and \( G^* = 0.7 \) (second harmonic mode of vortex shedding).

3.5. Interacting flow structures for \( G^* = 0.25 \)

For \( G^* = 0.25 \), the gap flow in Fig. 7 is stably and highly deflected toward the small cylinder. In the near wake region, the wake behind the large cylinder expands immediately into a wide wake; while that behind small cylinder is displaced to the free stream side. In Fig. 7(a)–(h), small waviness, denoted by \( A^* \), is observed to occur not so regularly along the upper shear layer. In Fig. 8(a), the velocity frequency spectrum, measured at \( X/D = 3.4 \), indicates that it is dominated by the frequency at \( f_{w0} \) accompanied by two small peaks centered at \( f_{w0} \) and \( f_{w0}/2 \). The spectrum in Fig. 8(b) shows that the spectral frequency of the narrow wake is \( f_{w0} \). At very small gap ratio (\( G^* = 0.25 \)), strong induction between two wakes indeed lead to very complicated flow structure. Thus, the velocity spectrum measured near the upper shear layer does not concentrate at \( f_{w0} \) (Fig. 8(a)) and the small waviness occurs not so regularly. This may be caused mainly by two factors. First, the amalgamation of the large vortex cluster is quite turbulent. Second, the separating shear layer of the wide wake may subject to intermittency during the developing and evolution. Several measurements are taken in the downstream region and their spectra show only a single spectral peak concentrated at frequency \( f_{w0} = f_{w0}/4 \). Instead of vortex pairing along the upper shear layer, the entrainment process of the small scale waviness into a
large-scale vortex is close to the “collective coalescence” of Ho and Huerre [6].

In Fig. 7(a)–(c), the counter-clockwise (CCW) vortices $B_1$ and $D_1$ coalesce first and then engulf the clockwise (CW) vortex $C_1$ into a large scale vortex ($B_1 + C_1 D_1$) in the near wake region. In Fig. 7(c)–(f), it takes about half of the period of the wide wake ($0.51 t^*$) to form the second large scale vortex ($B_2 + C_2 + D_2$). In Fig. 7(f)–(h), two vortex clusters ($B_3 + C_3 + D_3$ and $B_2 + C_2 + D_2$) further amalgamate together as a very large scale vortex cluster, shedding alternately downstream with that from the upper shear layer at a frequency $f_w^* \approx f_n^*/4$. In Fig. 7, the gap flow deflects stably toward the small cylinder and the narrow wake merges with the deflected gap flow shortly behind the small cylinder. The velocity spectra, measured in the far downstream region behind the cylinder couple, show only a single spectral peak centered at frequency $f_w^*$. This demonstrates that only a single wide wake at frequency $f_w^*$ is detected far downstream for $G^* = 0.25$ and $Re = 1000$.

In Fig. 7, the flow pattern far downstream is similar to that reported in Xu et al. [24] at $G^* = 0.3$, $Re = 350$ and in Lam et al. [15] at $G^* = 0.75$, $L^* = 0.0$ and Ko et al. [12] at $G^* = 0.4$, $L^* = 0.25$. 

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**Fig. 5.** Flow structures at selected instants within one complete cycle of the wide wake. Note $t^* = t/T_w$ and $t^* = 0$ is arbitrarily chosen. The normalized period of the wide wake is $1/f_w^* = 1.0$ and that of the narrow wake is around $1/f_n^* \approx 0.363$. The Reynolds number is 1000, $D/d = 2$ and $G^* = 1.25$. 

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However, the evolution and mutual interaction in the near wake are quite different in two respects at Re = 1000. First, along the upper shear layer, no pairing is observed or detected, and the small waviness is continuously entrained into a large-scale vortex. This process is close to the collective coalescence [5]. Second, the evolution of successive vortices B or the interaction scenario of vortices B, C and D are clearly revealed. As the gap ratio reduces, the downstream flow structures are observed to change from two vortex streets (e.g., one narrow and one wide wakes, Fig. 5 and 6) to one vortex street (Fig. 7). The critical gap ratio is around \( G^* = 0.25 \) which is about 20% smaller than \( G^* = 0.3 \) reported in the previous literature [3-8,11,23,24] for the side-by-side cylinders of equal diameter.

3.6. Spectral energy distributions and evolution for gap ratio \( G^* = 1.25 \)

Fig. 9(a) and (b) shows the distribution of the spectral amplitude for each dominant frequency over the flow domain. The symbols \( u = (f'_w) \) and \( u = (f'_n) \) represent respectively the non-dimensional spectral amplitudes at frequencies \( f'_w \) and \( f'_n \) based on Eq. (1). Here, \( f'_w \) and \( f'_n \) denote the characteristic frequencies of the wide and the narrow wakes, respectively.

In Fig. 9(a) for \( f'_w \) component, the maxima EA and EB are mainly concentrated at \( X/D = 3.8, Y/D = 2.5 \) and \( X/D = 3.4, Y/D = 1.6 \), respectively. This distribution represents a vortex street behind the large cylinder because the velocity fluctuations of EA and EB are nearly π out of phase (not shown here). In Fig. 9(b) for \( f'_n \) component, the local maxima ED and EC are concentrated at \( X/D = 1.4, Y/D = 0.4 \) and \( X/D = 1.6, Y/D = 0.2 \). This distribution also corresponds to a vortex street behind the small cylinder. Further, the contour distribution around EG coincides well with the evolution trajectory of vortices B (Fig. 5). In Fig. 9(a), contours with relatively small magnitudes at frequencies \( f'_w \) can be found in the downstream region along the upper shear layer of the small cylinder. Likewise, in Fig. 9(b), an extra contour region with relatively small magnitude around EG at \( f'_n \) exists near the lower shear layer of the large cylinder. Extension of the contour region of each spectral component behind other cylinder clearly indicates the mutual interaction between two wakes. In other words, the vortex shedding of one cylinder is influenced by the other one.

In Fig. 9(c)–(f), the maximum spectral amplitude of the local velocity fluctuations \( u'_w(f) \) are extracted from Fig. 9(a) and (b) along each separating shear layer. The non-dimensional elevation of the shear layer is defined as the location \( y_{c1,2} \) at which the local streamwise mean velocity equals \( (u_{max} + u_{min})/2 \) [18]. At each X/D station, the elevation of \( u'_w(f) \) in Fig. 9(c) and (f) coincide well with those passing through each local maximum EA, EB, EC and ED, respectively. According to the linear spatial stability theory, the maximum spectral amplitude of local velocity fluctuations \( u'_w(f) \) along each separating shear layer is defined in Eq. (2). Here \( k_x \) and \( k_y \) represent the real and imaginary parts of the complex wave number, respectively. Also, \( u'_w(f) \) is the spectral amplitude at \( X = 0, f \) is the characteristic frequency and \( t \) represents the time.

\[
u'_w(f) = |u'_w(f) + e^{i(2\pi k_x X + k_y Y)}| = u'_w(f) + e^{i\chi X}
\]

Thus, the streamwise variations of \( u'_w(f) \) at each dominant frequencies \( f'_w \) and \( f'_n \) in semi-log scale are employed to illustrate the spatial growth of the shear layer associated with the vortices A, B, C and D. In Fig. 9(c), the magnitudes of \( u'_w(f'_w) \) are smaller than that of a single cylinder of diameter \( D \) but the growing rates (or the slope) are nearly the same because the gap ratio (\( G^* = 1.25 \)) is large and the mutual interaction is relatively weak. In Fig. 9(d), the magnitudes of \( u'_w(f'_n) \) for vortex B are at least one order of magnitude smaller than those of \( u'_w(f'_w) \). In Fig. 9(e) and (f), the magnitudes of \( u'_n(f'_w) \) are smaller than within \( X/D < 2.0 \) and becomes the same.
in the region \(X/D > 2.0\) as those of a single cylinder of diameter \(d\). However, for the \(f_n^m\) component, the growing rates at initial stage are smaller than that of a single cylinder of diameter \(d\). For vortex \(C\) (Fig. 9(e)), the magnitude of \(u_m(f_m)\) overwhelms that of \(u_m(f_n^m)\) farther downstream because of the amalgamation of vortices \(B_1\), \(B_2\) and \(B_3\), and the weak entrainment of vortex \(C\) shown in Fig. 5. Besides, the onset locations of shear layer instability associated with each vortex are also examined. Since no data points with a level below \(10^{-2}\) are available behind the small cylinder, the level around \(10^{-2}\) is employed as a reference to evaluate the amount of delay \((Dx^*)\) of the onset locations of the shear layer instability. In Fig. 9(c)–(f), the onset of the shear layer instability for \(u_m(f_m)\) is delayed by an amount \(Dx^* = 0.5\); and that for \(u_m(f_n)\) is delayed by an amount \(Dx^* = 0.25\).

### 3.7. Spectral amplitude distribution for D/d = 2 at \(G^* = 0.75\) and 0.25

For \(G^* = 0.75\), Fig. 10(a) shows the distributions of spectral amplitude at \(f_n^m\) with the maxima (EB, EA) concentrated at \(X/D = 4.5\), \(Y/D = 1.0\) and \(X/D = 3.8\), \(Y/D = 2.2\). This distribution represents a wake with alternate vortex shedding behind the large cylinder. In Fig. 10(b), two maxima (ED, EC) of the spectral component...
Fig. 8. The velocity frequency spectra showing the interaction of the vortices depicted in Fig. 7. The Reynolds number is 1000, \(D/d = 2\) and \(G^* = 0.25\).

Fig. 9. (a and b) The dominant spectral amplitude distributions. The scale of \(u^*(f\omega)\) ranges from 0.0 to 0.3 and that of \(u^*(f n)\) from 0.01 to 0.17. (c–f) Spatial growth of \(u^*(f\omega)\) of the separating shear layer associated with each vortex for \(D/d = 2.0\) at \(G^*/Re = 1.25\) and \(Re = 1000\).
are asymmetrically concentrated at \( \frac{X}{D} = 1.1 \), \( \frac{Y}{D} = -0.3 \) and \( \frac{X}{D} = 2.0 \), \( \frac{Y}{D} = 0.2 \). This distribution corresponds to the narrow wake behind the small cylinder shown in Fig. 6. Adjacent to the maxima EC in Fig. 10(b), an additional contour around EG of reduced magnitude is found to closely follow the trajectory of the vortices B along which the mutual interaction between the vortices B and C takes place. For \( G^* = 0.25 \), Fig. 10(c) shows the local maxima (EA, EB) of \( u'(f_n) \) component at \( X/D = 8.0 \), \( Y/D = 2.0 \) and \( X/D = 6.5 \), \( Y/D = -0.4 \) in the far wake region. This corresponds to the single wide wake flow structure in the far wake region for \( G^* = 0.25 \). In Fig. 10(d), the two maxima (EC, ED) are concentrated at \( X/D = 1.5 \), \( Y/D = -0.1 \) and \( Y/D = -0.5 \), respectively. Behind the small cylinder, this highly tilted distribution corresponds to the narrow wake displaced by the severely deflected gap flow (Fig. 7). An additional contour (EG) adjacent to EC in Fig. 10(d), is caused by the amalgamation of vortices B, C and D shown in Fig. 7. Compared with those in Fig. 9(a) and (b) and 10, the magnitudes at both spectral components \( f_n^G \) and \( f_n^B \) decrease significantly as the gap ratio decreases. This clearly indicates that as the gap ratio reduces, the mutual interaction gets stronger.

### 3.8. Spectral amplitude distribution and evolution for \( D/d = 1 \) at \( G^* = 0.25 \)

To compare with the flow characteristics behind the side-by-side cylinders of equal diameter \( D/d = 1.0 \) at the smallest gap ratio \( G^* = 0.25 \), Fig. 11 shows a snapshot of the flow structure, the velocity frequency spectrum and the spectral amplitude distributions of two dominant components at \( Re = 1000 \). In Fig. 11(a), a very thin gap flow is observed to deflect toward the upper cylinder behind which a tilted closed narrow wake forms, leaving a thin separating shear layer in the region \( X/D > 2.0 \). Behind the lower cylinder, the lower separating shear layer is observed to roll up in the region \( X/D = 6.0–7.0 \). In Fig. 11(b), the velocity spectrum, measured near the end of the closed narrow wake region (\( X/D = 2.2 \)) of the upper cylinder, shows a dominant peak centered at \( f_n^G \). This spectrum clearly indicates that the high frequency component \( f_n^G \) is still detectable for the case of \( D/d = 1.0 \) and \( G^* = 0.25 \) (denoted by \( \oplus \) in Fig. 2(a)). In Fig. 11(c), the spectral amplitude at \( f_n^G \) is mainly distributed (denoted by EA, EB and EC) along the upper shear layer of the upper cylinder, corresponding to the amalgamation of the vortices A, B and C in the region of \( X/D > 2.0 \) depicted in Fig. 11(a). This amalgamation process along the upper shear layer of the narrow wake is basically similar to that of the vortices B, D and C in Fig. 7. Note that the local maxima of EA, EB and EC of \( u'(f_n) \) are much smaller in Fig. 11(c) than those in Fig. 10(d). This demonstrates that the influence on the vortex shedding of the narrow wake is more pronounced for \( D/d = 1 \) than for \( D/d = 2 \). This is also the reason why the high frequency component \( f_n^G \) is not detected (or difficult to measure) in the previous literature for the side-by-side cylinders of equal diameter spaced at \( G^* < 0.3 \). Fig. 11(d) shows the distribution of the spectral amplitude at \( f_n^G \) in which the maxima (EA, EB) are concentrated at \( X/D = 7.2 \), \( Y/D = 1.8 \) and \( X/D = 6.9 \), \( Y/D = -0.7 \).

### 3.9. Eigen functions and spatial growing along the shear layers

Fig. 12(a) shows the eigenfunctions of \( u*(f_n) \) in the near wake region and Fig. 12(b) illustrates those of \( u*(f_n^G) \) in the far wake region. In Fig. 12(a), for all the gap ratios studied, the magnitudes of \( u*(f_n^G) \) are dominant and those of \( u*(f_n^B) \) are negligibly small.
due to the early-stage development of the wide wake behind the large cylinder. Besides, the magnitude of $u \times (f'_w)$ decreases as the gap ratio reduces. Since the small cylinder is located at $Y/D = 0$, these double-peak distributions represent a wake behind the small cylinder. At the smallest gap ratio $G = 0.25$, one of the peaks is largely distorted because of strong mutual interaction of the vortices $B$ and $C$ shown in Fig. 7.

In Fig. 12(b), the eigenfunctions $u \times (f'_w)$ take the dominance and also exhibit double-peak distributions. For the gap ratios $G = 1.25$ and 0.75, the distributions are similar but shift a distance about 0.5D downwards because the large cylinder is situated closer to the small cylinder. For $G = 1.25$ and 0.75, the magnitudes of $u \times (f'_w)$ are smaller than but comparable with those of $u \times (f'_w)$ (not shown here). Coexistence of the eigenfunctions $u \times (f'_w)$ and $u \times (f'_w)$ in the far wake region clearly indicate that the flow farther downstream is a two vortex-streets structure for $G = 1.25$ and 0.75. These flow structures are similar to that reported by Xu et al. [24] for side-by-side cylinders of equal diameter. For $G = 0.25$, the distribution of $u \times (f'_w)$ shifts about 1.0D downwards and spreads much wider in transverse direction covering both cylinders. In the far wake region, the magnitudes of $u \times (f'_w)$ becomes negligibly small for $D/d = 2$ at $G = 0.25$. This indicates that a single wide wake at frequency $f'_w$ has already formed downstream of this location. Besides, in Fig. 12(b), the distribution of Eigen-function $u \times (f'_w)$ for $D/d = 1$ at $G = 0.25$ resembles that of $D/d = 2.0$ except the differences in magnitudes. This again represents a single wide wake behind the cylinder couple as depicted in Fig. 11(a).

In Fig. 12(c), the spatial evolutions of $u_m \times (f'_m)$ along the shear layer associated with the vortex $A$ are compared in the semi-log scale. For a single cylinder of diameter $D$, the amplitude of $u_m \times (f'_m)$ grows almost linearly at initial-developing stage, reaches the maximum value near $X/D = 3.0$ and then decreases monotonously downstream. For side-by-side cylinder of $D/d = 2$ at $G = 0.75$ and 1.25, the developing curves are not only parallel to that of a single cylinder of diameter $D$ but also exhibit an amount of delay ($Dx = 0.5$). The maximum values of $u_m \times (f'_m)$ occur at around $X/D = 3.5$ which is about 0.5D downstream that of a single cylinder of diameter $D$. At the smallest gap ratio $G = 0.25$ for $D/d = 2$ and 1, the developing curves are very close to each other but with much smaller slopes than those of $G = 1.25$ and 0.75. Namely, the spatial growing rates for $G = 0.25$ is much smaller than those of $G = 1.25$ and 0.75.

In Fig. 12(d), the spatial evolutions of $u_m \times (f'_m)$ along the shear layer associated with the vortex $D$ are compared in the semi-log scale. For a single cylinder of diameter $d$, the developing curve of $u_m \times (f'_m)$ also exhibits linear growth at the initial developing stage. It reaches the maximum value near $X/D = 1.0$ and then decreases monotonously downstream. For side-by-side cylinder of $D/d = 2$ at $G = 0.25$, 0.75 and 1.25, all the developing curves show linear growth at the initial developing stage. As the gap ratio decreases, the slope increases slightly; the maximum delay is $Dx = 0.5$ and the locations of maximum $u_m \times (f'_m)$ move about 0.5D downstream compared with that of a single cylinder with diameter $d$. However, for $D/d = 1$ and $G = 0.25$, the magnitude of $u_m \times (f'_m)$ at the linear developing stage is negligibly small and the amount of
delay \((Dx^*)\) is significant compared with that of \(G = 0.25\) and \(D/d = 2\).

3.10. Discussions

The vortex formation length is defined as the streamwise distance between the cylinder center and the location where the spectral amplitude of velocity fluctuation reaches the maximum. The shorter the vortex formation length, the lower is the base pressure behind the cylinder. Thus, the form drag of the cylinder will be enhanced at shorter vortex formation length. Besides, for a single cylinder, the shorter the vortex formation length, the higher is the vortex shedding frequency. However, for side-by-side cylinders, one cylinder indeed influences the shear layer development and evolution of the other cylinder. Thus, the vortex formation length may be closely related to the combination effects of the spatial growth rate and the delay of the onset location of the shear layer instability. In the following, the discussions will be focused on the spatial growth rate and the delay of the onset location of the shear layer instability at various gap ratios and diameter ratio.

While a cylinder is placed side-by-side close to another cylinder at reducing gap ratio, two flow characteristics are modified relative to that of a single cylinder. First, the onset locations of the shear layer instability. Second, the spatial growing rates of the shear layers. In this study, as the Reynolds number is kept constant, these two modifications depend upon the gap ratio and the diameter ratio.

The spatial growth rate \((k_i = 2\pi f_i U)\) is proportional to the characteristic frequency of the shear layer instability. At a fixed Reynolds number, the velocity along the shear layer can be assumed nearly constant. Thus, small spatial growing rate (or slope) implies low characteristic frequency. In Fig. 12(c), as the gap ratio reduces, the spatial growing rate of the outer shear layer of the wide wake decreases mildly at \(G = 1.25\) and \(0.75\) and significantly at \(G = 0.25\). This corresponds to slow variations of \(St_w\) at large gap ratio and rapid change of \(St_w\) at small gap ratio in Fig. 3. On the other hand, in Fig. 12(d), the slope of \(u_m(t^*_w)\) component increases slightly as the gap ratio decreases. So do the values of \(St_n\) increase as the gap ratio decreases in Fig. 3. For \(D/d = 2\) in Fig. 12(c) and (d), as the gap ratio decreases, the reduced spatial growing rate of the shear layer of wide wake is more significantly than the increasing spatial growing rate of the narrow wake. This explains the rapid increase of the frequency ratio at small gap ratio shown in Fig. 4 for \(D/d = 2\).

In Fig. 12(c) for \(G = 1.25\) and \(0.75\), the slopes are about the same as that of a single cylinder with a noticeable delay \((Dx^*)\). Thus, for \(D/d = 2\) at \(G = 1.25\) and \(0.75\), the vortex formation length becomes \(0.5D\) longer than that behind a single cylinder of diameter \(D\). In Fig. 12(d) for all gap ratio studied, the maximum delay is about \(0.5D\) and the location of maximum spectral amplitude also shifts \(0.5D\) downstream for \(G = 0.25\) relative to that of a single cylinder of diameter \(d\). The locations of maximum

Fig. 12. The Eigen-functions measured (a) in the near wake for the spectral component \(u(t^*_m)\), (b) in the far wake for the spectral component \(u(t^*_w)\) for the side-by-side cylinders at \(Re = 1000\). (c) Spatial growth of \(u_m(t^*_w)\) of the separating shear layer associated with vortex A (wide wake) and (d) spatial growth of \(u_m(t^*_n)\) of the separating shear layer associated with vortex D (narrow wake).
spectral amplitude coincide with the local maxima (EA, EB, EC and ED) shown in Figs. 9 and 10. Based on the results in Fig. 12(c) and (d), the maximum locations of the spectral amplitude are affected by both the growing rate and the amount of delay \(Dx/\). In Figs. 9 and 10, ratios of the averaged streamwise distance of the maxima EA (or EB) and EC (or ED) are about 2.57 (or 3.6/1.4) for \(G^\ddagger = 1.25\), about 2.87 (or 4.3/1.5) for \(G^\ddagger = 0.75\) and about 4.5 (or 7.2/1.6) for \(G^\ddagger = 0.25\). These ratios are very close to the frequency ratios of 2.56 for \(G^\ddagger = 1.25\), 3.1 for \(G^\ddagger = 0.75\) and 4.2 for \(G^\ddagger = 0.25\) shown in Fig. 4 for \(D/d = 2\).

3.11. Some turbulent statistics

Fig. 13 illustrates the distributions of \(u_{rms}\) and \(uv^\prime\) over the flow domain for side-by-side cylinders of diameter ratio two at \(G^\ddagger = 0.25\)–1.25. In Fig. 13(a) and (c), the locations of local maxima GA and GB are well defined and differ only slightly from those in Figs. 9 and 10(a) and (b) because of relatively large gap ratio. So do the local maxima GD and GC. They describe the turbulent statistics of \(u_{rms}\) of the wide and narrow wakes, respectively, at \(G^\ddagger = 1.25\) and 0.75. At the smallest gap ratio in Fig. 13(e), the locations of GA and GB deviate largely from that depicted in Fig. 10(c). Compared with those of \(u_{rms}\) (left column of Fig. 13), the local maxima of \(uv^\prime\) (right column of Fig. 13) distribute symmetrically on both sides of each cylinder. The distribution of \(uv^\prime\) considers the time-averaged turbulent statistics for two velocity components for all spectral components; but that of \(u_{rms}\) demonstrates the turbulent statistics in the root-mean-square sense for only one velocity component. To elucidate the evolution of each shear layer with dominant frequencies, it is better to employ the spatial variations
of \( u_n(f'_n) \) and \( u_n(f'_w) \) for each spectral component shown in Figs. 9 and 12.

4. Concluding remarks

Dye flow visualization and particle image velocimetry (PIV) are employed to study the vortex interaction scenarios and the downstream flow patterns behind side-by-side cylinders of diameter ratio two at different gap ratios. Velocity measurements are made and analyzed at \( Re = 1000 \) for gap ratios 1.25, 0.75 and 0.25. The variation trends of Strouhal numbers are measured at \( Re = 1000, 2000, 5000 \) as functions of the gap ratio and diameter ratio.

For the Reynolds numbers studied, as the gap ratio increases, the Strouhal number of the narrow wake decreases monotonously but that of the wide wake increases also in the monotonous way. The gap flow is always stably deflected toward the small cylinder. The frequency ratio \( f'_n/f'_w \) depends strongly upon the gap ratio and the diameter ratio; but is nearly independent of the Reynolds number studied. For side-by-side cylinders of diameter ratio two and \( Re = 1000 \), two different vortex interaction scenarios are found for \( G = 0.25 \) and 0.75 \( \leq G \leq 1.25 \). The former leads a single vortex-street flow structure in the far wake region; the latter results in one narrow and one wide wakes with different characteristic frequencies. The critical gap ratio is around \( G = 0.25 \) for \( D/d = 2.0 \) which is slightly smaller than that \( (G = 0.3) \) reported for the counterpart of equal diameter.

While a cylinder is placed side-by-side close to another cylinder at reducing gap ratio, two flow characteristics are modified relative to that of a single cylinder. First, the onset locations of the shear layer instability. Second, the spatial growing rates of the shear layers. As the gap ratio decreases, the reduced spatial growing rate of the wide wake is more significant than the increasing spatial growing rate of the narrow wake. The locations of maximum spectral amplitude are affected by both the growing rate and the amount delay \( (\Delta t^*) \) of onset of the shear layer instability. The frequency ratio of the narrow and the wide wakes can be related by the ratios of the averaged streamwise distance of the local maxima of the spectral amplitudes.

For the side-by-side cylinders of diameter two \( (D/d = 2) \), the influence on the wide wake is more significant but is less pronounced on the narrow wake. Further, the influences on both the narrow and the wide wakes are even pronounced for \( D/d = 1 \) than those for \( D/d = 2 \).

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