A THRESHOLD MODEL TO EXPLAIN AND PREDICT ECONOMIC DECISIONS

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Introduction

Many economic decisions may be characterized as having a threshold level below which a stimulus elicits no observable response. When the strength of the stimulus reaches the threshold level a reaction occurs. Any additional increase in stimulus strength beyond the threshold point results in no further response. Explaining and predicting these kinds of economic decisions and behavior of firms or individuals requires specialized models to identify the relevant economic stimuli, to provide information on the magnitude of their effects, and to estimate the threshold levels of responses.

Although a number of studies have been made of the economic decision processes [15; 13; 20; 17], none have adequately integrated the theory of the threshold concept and the operational statistical techniques into an economic model. Nourse and Berry [18; 2] referred to the concept in their studies of market areas and centers, but applied the term only to the problems of minimum size of firm. Bilkey [3] in a study of consumer expenditures implied the threshold concept in his vector analysis although he did not explicitly develop a threshold theory or attempt to measure the threshold levels. Models developed by Tobin, Rosett, and Dagenais [22; 19; 5] can be used to estimate parameters for dichotomous problems but no explicit connection was made among decision theory, the threshold concept, and the statistical techniques. These models were focused primarily on limited regressand problems.(1) Tomek [23] discussed the dichotomous regressand problem and related statistics, but like Tobin and others did not relate his study to the theory of threshold reactions. In contrast, modern psychologists such as

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Isaacson, Heckel, and Blum [12; 9] emphasize the important relationship between the threshold concept and human decisions or behavior, but their approaches are mainly conceptual rather than operational and quantitative.

Integration of the theoretical and statistical models is important in explaining and predicting dichotomous decisions and behavior. The objectives of this paper are: (1) to briefly develop the threshold concept as it relates to economic decisions, (2) to show a generalized statistical model apprpriate for estimating the relationships between the economic stimuli and a dichotomous decision response, and (3) to illustrate the model through an application to decisions of farmers to purchase grain dryers.

Theoretical Framework of the Madel

Economic decisions are frequently dichotomous. For example, a decision is made to either accept or reject a proposed program or policy; a business firm decides either to make or not to make a particular investment; a consumer decides either to purchase or not to purchase a specific commodity such as an automobile or TV set, during a particular period, etc. In situations where an economic unit is forced to choose among several alternatives rather than between two, the problem can still be formulated as a dichotomous decision with respect to each alternative.

The dichotomous nature of a decision implies that there exists a "breaking point" in the dimension of the explanatory variable such that a positive decision will be made in any portion of the range above this crucial point. If the observed decision variable, Y, is defined in the context of probability, then Y will take only two values: $Y=1$ if the decision results in an action and $Y=0$ if it results in no action, The dichotomous nature of this kind of decision is illustrated by the line ATUB in Figure 1. The point T represents the "breaking point" which, in terms of psychology or biology is usually called the "limen" or "threshold," At values of X greater than T there is a probability of one of a positive action or decision. At values below T the probability is zero.

The concept of a threshod is commonly recognized in biology, where it is found that there exists a response-threshold for most biological organisms. That is, a certain level of stimulus strength is necessary before a response or reaction to the stimulus will take place. For example, an insect will be killed only when the application of insecticide is beyond a certain level but it will not be killed by any lower level of dosage [7]. This minimal level of dosage which stimulates the response is the "threshold" of that particular insect. However, it is important to note that the threshold level, in general, is not identical for all individual insects of the same kind.

A similar phenomenon can be observed in economic decisions or behavior. An economic unit will not respond to a change in a particular economic variable below its threshold level [3]. A small increase in famity income, for example, may have no effect on the existing consumption pattern of that household; or a change in the price of a product may not alter its effective demand until the magnitude of change exceeds a certain level, say. 5 percent of the price, etc. The threshold concept therefore, is an essential element in explaining many economic decisions and behavior processes.

As pointed out previously, if the decision variable, Y, is dichotomous (i. e., takes only a lower value of O and an upper value of 1) and the thresholds of all members in a sample are not identical. (e. g., T_1, T_2, T_3,……T_n in Figure 2), then the "expected aggregate relationship" of the threshold decision model would resemble a "sigmoid curve." Moreover, if the distribution of these thresholds is assumed to be normally distributed, the expected aggregate relationship will be a "normal sigmoid curve" such as the AB curve in Figure 2. The main characteristic of a normal sigmoid curve is that there exists sections in the lower and upper ranges of the explanatory variable (or variables) in which an increase in the explanatory variable would not exert any (or at most a negligible) incluence on the value of the decision variable. A sensitive response in the decision variable can be observed only in the segment between the two extremes [7, pp. 8—47]. The end-point of the lower range may be regarded as the "threshold" of that explanatory variable. Beyond the threshold point, a normal sigmoid curve first increases at an increasing rate and then increases at a decreasing rate, finally reaching a maximum at a probability value of 1. Therefore, it can be seen that the sigmoid curve represents the functional relationship of a threshold model in which the decision variable is dichotomous. Additional discussion of the statistical properties of the threshold model presented in this study will be given in the following sections.

Methodological Considerations

The discussion and illustration in the previous section emphasizes two important points. First, economic decisions are often dichotomous and the level of the "breaking point" (threshold) of each individual may not be the same. Second, if the decision variable (used as a dependent variable) is dichotomous and the thresholds of individual members are not necessarily identical, the functional relationship between the decision variable and explanatory variables must be of a sigmoid type. The methodological task therefore, is to find an estimating procedure capable of representing these relationships.

The technique most commonly applied to this dichotomous dependent variable problem has been the linear probability function model [8, p. 249] which, in fact, is an ordinary regression technique with a O or 1 regressand. Unfortunately, there are several weaknesses associated with this approach.

(1) The threshold concept is not incorporated in the model.

(2) The disturbance term exhibits heteroscedasticity. Assuming a linear model $Y_i = X_i'\beta + U_i$ in which X_i' is a row vector of regressors and β is a column vector of parameters, then, it is easily seen that:

$$\begin{aligned}\mathrm{Var}\ (U_i) &= E[U_i - E(U_i)]^2 = E(U_i^2) - [E(U_i)]^2 \\ &= E(U_i^2) - 0^{(2)} \\ &= (1 - X_i'\beta)^2 (X_i'\beta) + (-X_i'\beta)^2 (1 - X_i'\beta)^{(3)} \\ &= (1 - X_i'\beta)(\iota'\beta)\end{aligned}$$

Thus, the variance of U_1 is heteroscedastic varying according to each different X_i, and the ordinary assumption of homoscedasticity is violated. Although the least-squares estimator is still unbiased, it is no longer efficient.

(3) The expected value of Y_i may sometimes be greater than 1 or less than 0 [8, p. 250], i. e. $1<E(Yi)<0$. This is probably a more serious problem than that of heteroscedasitc disturbances because it is inconsistent with the definition of the dichotomous dependent variable and the interpretation of the expectation of Y as a probability.(4)

(4) The functional form of the model is limited to a linear relationship between the dependent and independent variables. As discussed previously, the expected functional form of a dichotomous threshold problem is a sigmoid curve. The linear probability function model can only approximate this non-linear relationship with a linear function. Therefore, the accuracy of the estimator of the model is acceptable only over a limited segment of the sample observations [22, p. 36].

To overcome the heteroscedasticity problem, one may employ either a transformation procedure or Aitken's generalized least squares. However, further difficulties exist in both alternatives. In the transformation procedure, the weighting factor

$\frac{1}{\sqrt{E(Y_i)[1-E(Y_i)]}}$ [14, p. 208] would be indeterminable if $E(Y_i)>1$ or $E(Y_i)<0$ and the procedure breaks down. The Aitken's generalized least squares technique is applicable only under the condition that the variance-covariance matrix (Ω) is nonsingular so that Ω^{-1} is obtainable.

In order to handle the problem of $1<E(Y_i)>0$ and the problm of linearity, various models such as logit [1], gompit [4], probit [7], and Zellner and Lee's joint-estimation technioue [25], have been developed. However, only the probit model can also incorporate the threshold while approximating the dichotomous decision type of functional relationship. Probit therefore provides an acceptable answer to all four of the problems discussed in the preceding paragraphs. The characteristics of the probit (or normit) model will be discussed in more detail in the following section.

The Statistical Model(5)

Single Variable Case

The statistical application of the threshold concept can be most easily seen in a simplified case in which the decision variable is assumed to be a function of only one explanatory variable. Let Y= the decision to purchase a particular durable good (e.g. color television set), X= dis posable family income, and X^*= the income-threshold with respect to the purchase of a color television. Then, according to the theories and illustration discussed previously, the ith consumer will decide to make the purchase only under the condition that the value of X_i is equal to or greater than X_i^*, and it will not be purchased if the value of X_i is smaller than the threshold value X_i^*. If the decision variable Y is defined in terms of probability, the value of the observed Y would be determined as follows:

$$(1) \qquad Y_i=\begin{cases}1, & \text{if } X_i \geq X_i^* \\ 0, & \text{if } X_i < X_i^*\end{cases}$$

From equation (1), it is clear that X^* represents the "breaking point" (threshold) mentioned above and plays the role of determining the purchasing decision in the model. It is conceivable that some consumers have a relatively high X^* so that they would not purchase a color television set until their income becomes very high. Other consumers may have a relatively low X^* so that they might purchase the product even though their income is lower than that of the previous group. The reason that each X_i^* is not necessarily identical may be attributed to individual differences such as personality, preferences and taste, educational background, and philosophy of value, etc. Although X^* is an unknown random variable whose parameters are to be estimated, the distribution of these X_i^*'s based on the central limit theorem. can be assumed to be normal [6, p. 178]. This normality assumption about the distribution of thresholds is the essential and indispensable idea in all probit approaches.

In reality, the decision to purchase a durable good is seldom a function of only disposable income but is, in general, influenced by many additional variables such as the age of the head of the household, the size of the household, exposure to advertising, the past habits and experiences of the consumption unit, etc. This implies that a multivariate model is more appropriate in most problems of estimation.

Multi-stimuli case

Let the decision variable Y be hypothesized to be a function of mexplanatory variables $(X_1, X_2, \cdots\cdots X_m)$ where $X_1=1$ for all i. Then, in order to incorporate the threshold concept into the model, assume there is an aggregated variable, A, such that A is a linear combination of X's. Or, in an explicit algebraic form:

$$(2)\qquad A_i=\beta_1 X_{1i}+\beta_2 X_{2i}+\cdots\cdots+\beta_m X_{mi}$$

It should be noticed that, since these X's can take logarithmic, exponential, or any other nonlinear form, the assumption of a linear combination does not restrict equation (2) to a linear function and any nonlinear functional form can still be presented. Let A^* be the threshold value of A analogous to the X^* in the single-stimulus model, then by a parallel analysis, equation (1) can be rewritten as:

$$(3)\qquad Y_i=\begin{cases}1, & \text{if } A_i\geq A_i^* \\ 0, & \text{if } A_i<A_i^*\end{cases} \quad \text{for all } i,\ i=1, 2, \cdots n$$

Since A^*, which is assumed to be normally distributed, can be standardized to be a random variable with N(0,1), the probability that $Y=1$, given A, would be:

$$(4)\qquad \text{prob}\ (Y=1/A)=\text{prob}(A\geq A^*/A)=F(A)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\beta_1X_1+\beta_2X_2+\cdots\cdots+\beta_mX_m} e^{\frac{-(A^*)^2}{2}}\,dA^*$$

and the probability that $Y=0$, given A, would be:

$$(5)\qquad \text{prob}\ (Y=0/A)=\text{prob}\ (A<A^*/\text{A})=1-F(A)=\frac{1}{\sqrt{2\pi}}\int_{\beta_1X_1+\beta_2X_2+\cdots\cdots+\beta_mX_m}^{\infty} e^{\frac{-(A^*)^2}{2}}\,dA^*$$

where $F(t)=$ the value of standard normal cumlative function evaluated at point t.

The statistical task is to find the estimates of β's such that the substitution of these estimated values of β's in the above equations will come as close to the observed values of Y as possible. From equations (4) and (5), it is clear that the probabilities for $Y=1$ and $Y=0$ are both a function of β's. Therefore. it is possible to obtain the maximum likelihood estimates (MLE) of β's which satisfy the condition specified above if the likelihood function of the sample can also be defined as a function of β's.

Let n be the sample size and r the total number of observations with an observed value of $Y=1$. The total number of observations with $Y=0$ is thus n-r. For convenience, rearrange the order of the observed sample such that the first r observations have $Y=1$ and the remaining n-r observations have $Y=0$. Then, if these observations ars assumed to be drawn in independently of others, the likelihood function of the sample can be defined to be a function of β's as follows:

$$(6)\quad L=[F(A_1)]\,[F(A_2)]\cdots\cdots[F(A_r)]\,[1-F(A_{r+1})]\,[1-F(A_{r+2})]\cdots\cdots[1-F(A_n)]$$

Or, in logarithmic terms:

$$(7)\quad L^*=\log(L)=\sum_{i=1}^{r}\log[F(A_i)]+\sum_{i=r+1}^{n}\log\,[1-F(A_i)]$$

Finding the maximum likelihood estimates of the β's is identical to finding values of β's which maximize this likelihood function.

Taking partial derivatives of equation (7) with respect to each $\beta_j (j=1,\ 2,\ \cdots m)$ and setting them equal to zero, a system of m equations (the normal equations) can be obtained which are of course nonlinear. The maximum likelihood estimates of the β's are obtained by solving this set of normal equations through some interative process techniques [21, p. 7].

The conditional expectation of Y_i given A_i is defined as:

$$(8)\quad E(Y_i/A_i)=Y_i f\,(Y_i/A_i)$$

where f (Y_i/A_i) is the conditional probability density function of the corresponding Y_i which takes a value of either 1 or 0. In cases where $Y_i=0$, the $E\,(Y_i/A_i)=0.\text{f}\,(Y=0_i/A_i)$ so the $E\,(Y_i=0/A_i)$ is equal to zero. If $Y_i=1$, then the $\text{E}\,(Y_i/A_i)=1.\text{f}\,(Y_i=1/A_i)=\text{prob}\,(Y_i=1/A_i)$. Therefore, the conditional expectation of Y_i in the probit model can be defined as:

$$(9)\quad E(Y_i/A_i)=\text{prob}\,(Y_i=1/A_i)=F(A_i)$$

It is evident from equation (9) that the conditional expectation of Y_i in the probit method is the estimated probability that the particular decision in question will be mad.

After having obtained the MLE of β's, they may be inserted in equation (9), and the probability that a particular economic decision will be made can be derived from the following equation:

(10) $\hat{Y}_i = F(\hat{A}_i) = F(\hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \cdots\cdots + \hat{\beta}_m X_{mi})$

To test the statistical significance of these coefficients, one may employ either the standard t-test or the likelihood-ratio method suggested by Tobin. The standard t-test is more convenient when testing the significance of an individual coefficient such as $H_0 : \beta_i = 0$ or $H_0 :\ \beta_i = \beta_j$.

The likelihood-ratio method can be used to test the coefficients individually (as above) or jointly. If a joint test is made of the null hypothesis $H_0 :\ \beta_2 = \beta_3 = \cdots\cdots = \beta_m = 0$, it would indicate whether the economic decision, Y, is independent of the X's. Therefore, it can be considered as a test of the appropriateness of the model in explaining the observed phenomenon.

The likelihood-ratic test uses the statistic [(-2) log (λ)], where $\lambda$is the ratio of the logarithms of two likelihood functions: (1) a likelihood function evaluated by setting all coefficients corresponding to each explanatory variable equal to zero except the estimated constant coefficient, $\hat{\beta}_1$,--i.e. $L(\hat{\beta}_1, O_2, O_3 \cdots\cdots O_m)$; and (2) a likelihood function evaluated with all estimated coefficients free of any restriction. For large samples when the hypothesis is true, the statistic [(-2) log λ] approximates a chi-square distribution with $(m-1)$ degrees of freedom, where m = the number of unknown parameters to be estimated in the system [16].

Properties of the Model

The multivariate probit model as described above has several important statistical and methodological properties.

1. From equations (1) and (3), it can be explicitly seen that the threshold theory and concept has been incorporated in the model to determine the economic decision. Most dichotomous regressand models (such as the linear probability function model) either ignore the threshold concept or are unable to explicitly incorporate it into the model.
2. The problem of heteroscedosticity is eliminated in this probit approach by the assumption of normality in the distribution of A^*. The decision variable, Y, is no longer a

direct function of the explanatory variables but a function of A and A^* which plays the role of disturbance forces in the model.

3. From equation (9), it is evident that the expected value of Y_i is confined to the range between 0 and 1. Thus, the difficulty of $1 < E(Y_i) < 0$ mentioned on page 6 can be overcome in the probit model.
4. Furthermore, equation (2) and (9) imply that the functional form derived under probit approach is a normal sigmoid curve [7]. Therefore, the weakness of linearity discussed in the previous section is removed and the typical relationship of the threshold theory is achieved.
5. This model implies that the magnitude of the effect from the relevant economic stimuli depends upon the status of the economic unit and not solely upon the magnitude of the estimated coefficient, $\hat{\beta}_j$. (6) To state this more rigorously, $\Delta E\ (Y_i)$ is a function of A_i and ΔA_i where $\Delta A_i = \hat{\beta}_j\ (\Delta X_{ij})$. For example, an increase of a certain percent of the ith cosumer's disposble income, ΔX_{ij}, will increase his probability of making a particular purchasing decision. The amount of increase in probability is determined not only by the magnitude of $\hat{\beta}_j$ (as in ordinary regression), but also by the initial value of A_i before the change of X_{ij} takes place. Therefore, the effect of a one percent change in disposable income also depends on the initial income level of that particular econmic unit. This is consistent both with economic theory and with empirical observation.

From the preceding discussion, it is evident that in problems where the dependent variable is dichotomous and a threshold phenomenon or a sigmoid relationship is assumed, the application of this multivariate probit model is preferable to other statistical techniques.

An Illustrative Application

During the past decade there has been a number of changes in the production and marketing of corn due to the shift from ear corn harvesting to field shelling. None has been more dramatic than the expansion of drying capacity on farms and at country elevators. Despite the economies of scale favoring drying and storage at country elevators, on-farm drying has expanded by 385 percent compared to an 88 percent expansion in drying at country elevators (10, p. 7). It is important for dryer manufacturers and elevator managers to understand these trends as they try to assess the market potential for their respective product or service, and to identify the specific characteristics of individuals or the marketing strategies that influence the decision to use farm dryers or elevator drying services.

The primary advantage of field shelling is realized only when corn is harvested at moisture levels above the maximum limit for safe storage. High moisture shelled corn must be dried within a few days of harvest to prevent quality deterioration. Farmers in general have two alternatives from which to choose: 1. Deliver the corn to the elevator where it is dried at a known rate per bushel[7] or, 2. purchase a dryer and other equipment necessary for drying the corn on the farm. The purchase of a grain dryer and the associated handling and testing equipment for on-farm drying represents a major capital investment that is usually depreciated over 10 to 20 years. The decision to make the purchase can best be characterized as a dichotomous decision. The threshold concept provides a basis for describing, explaining and predicting this decision by individual farmers or groups of farmers. when the cumulative effect of the economic and psychological factors of a farmer become sufficiently large, a dryer is purchased. Changes above or below this threshold result in no visible action or decision.

The economic feasibility of the purchase depends on the relative profitability of the alternative uses for resources as well as on the cost of drying. The dichotomous choice between on-farm and off-farm drying is therefore influenced by the economic organization of farm enterprises and by the characteristics and attitudes of the individual farmers.

The economic decision model includes the variables of farm size, farm type, method of harvest, farm ownership, shelled corn storage capacity, percent of grain sold at harvest and age of the farmer; each of which is related to the purchase decision through a threshold concept. [8] For example, changes in farm size below the threshold level will have no observable effect on the purchase of a dryer. Once the size is increased to the threshold, a dryer in purchaed and further increases in size may affect the capacity of the dryer, but not the decision to purchase a dryer.

Since decisions and decision processes are internalized and not directly observable, the action resulting from the decision (i. e. the ownership of a dryer) was assumed to be equivalent to the decision and was used as the dependent variable in the model. Due to the cross sectional limitation of the data, no dimension of time was incorporated.

The size of enterprise is related to the decision through the economies of scale in operation of a grain dryer. The type of farm, method of harvest, shelled corn storage capacity, and percent sold at harvest are all related to the disposition or use of the corn and the costs associated with the alternatives. Hog farms use primarily shelled corn that must be dried prior to storage. Grain sold at harvest is usually sold without drying. Corn cannot be safely stored at moisture levels above 15 percent so the use of shelled corn storage depends on dring capacity on the farm. Corn dried at the elevator and returned to the farm for feeding or storage requires trucking at charges that generally exceed the cost of drying unless the volume of corn is very small.

Farm storage and drying facilities are long-term permanent investments requiring some assurance of security of tenure. Owner operators are therefore in a better position to make these investments than tenants. Due to limitations of equipment and storage capacity, many farmers use a combination of ear and shelled corn harvest. Ear corn is seldom dried artificially so the percent of corn field shelled is sn important determinant in the decision to purchase a farm dryer. Due to the technical knowledge required in field shelling and drying corn, age and education are also important (though highly intercorrelated) variables. The length of time required for recovering the investment in shelled corn harvesting, drying, and storage facilities also discourages older farmers from buying a dryer.

The variables and their definitions are shown in Table 1. Data on these variables were obtained by the Farm Research Institute, Urbana, Illinois, from their randomly selected panel of Illinois farmers. An IBM/360-75 was used to run the multivariate probit program using these data. The results are summarized in Table 2.

The hypothesis that $\beta_2 = \beta_3 = \cdots\cdots \beta_8 = 0$ was tested using the likelihood ratio statistic described previously. The calculated X^2 value of 367.36 (Table 2) results in rejection of the null hypothesis at $\alpha = .01$; suggesting that this set of variables is relevant in the explanation of a farmer's decision to purchase a dryer.

The above joint test of the vector of coefficients does not assure that each coefficient is statistically significant. To test the hypotheses that $\beta_j = 0$ ($j = 1, 2, \cdots\cdots 8$), the standard t-test was applied to each individual coefficient. All of the coefficients except β_5 were significantly different from zero at $\alpha = .01$: β_5 was significant at $\alpha = .05$.

The signs of all the estimated coefficients were consistent with hypothesized relationships. An increase in farm size, percent of corn field shelled, and farm storage capcity results in an increase in the probability of purchasing a grain dryer. A farm producing hogs has a higher probability of purchasing a dryer than other farm types. The probability of dryer ownership is decreased by an increase in the percent of corn sold at harvest, and by an increase in the average age of the farm operator. A farm operated by a tenant has a lower probability of purchasing a dryer than one operated by the owner.

The magnitude of the effect of each variable on the probility of purchasing a dryer is only indirectly related to the size of the coefficients, owing to the indirect procedures of the probit technique in estimating probability. It is evident from equation (10) that given a set of values for all X's, the magnituae of $\hat{\beta}$'s determines the value of the index for the i-th farmer ($\hat{A}_i$). The expected value of Y_i (i. e. the probability of dryer ownership by the ith farmer) is determined by the value of that $\hat{A}_i$ through the normal cumulative function.

For example, assume for simplicity that the decision to purchase a dryer is a function of only the percent of corn field shelled, X_4. Since $\hat{\beta}_4 = .0147$, an increase of ten percentage-points in the percent of corn field shelled does not increase $\hat{Y}_i$ by .147 but increases the value of $\hat{A}_i$ by .147. The increase in probability, $\Delta\hat{Y}_i$, resulting from the increase in $\hat{A}_i$ depends on the level of the initial value of $\hat{A}_i$ existing before the change in field shelling takes place. If the initial level of $\hat{A}_i$ is very high or very low, then a ten percentage-point change in X_4 will exert little influence on the probability. If the initial level is near the "threshold" of the farmer, the increase of .147 in $\hat{A}_i$ will have a significant effect on the probability. The relationship between A and the probability can be found in the Probit-Probability Transfomation Table [9] The relationship is also illustrated by Figure 2.

Given these probit maximum likelihood estimates and a set of values of X's, the probability that a grain dryer will be purchased can be estimated from the final equation derived in the previous section. For example, if a farm has 500 hogs or more, has planted 500 acres of corn, field shells 80 percent of total production and sells 20 percent directly from the field, and if the farmer is a renter-operator 40 years of age, the calculated probability that this farmer will purchase a dryer is approximately .76. Furthermore, if the mean-values of these X's for a particular market or region are known, the potential sales in this market can be predicted from these estimated probabilities. The same analogy can be easily extended to estimate sles for a total area where several different market segments exist.

A second model was analyzed using the same program and variables, except that "intentions to purchase a dryer" was substituted as the dependent variable, and "present dryer ownership" was included as an additional independent variable. The results of this model are shown in Table 3. The significance (and lack of significance) of individual coefficients in this model provides additional insight into the economic stimuli affecting the decision to purchase a farm dryer.

Although the variable "farm shelled corn storage capacity" was significant in the previous model, the direction of causality could not be determined. The lack of significance of this variable in the intentions model suggests that it does not enter into the planninig decision for dryer purchases. The majority of farm dryers include a storage bin with a heating unit for drying. Additional storage is purchased separately. Thus, purchases of storage and drying facilities are joint (often simultaneous) decisions, and availability of shelled corn storage is not a prerequisite for purchase of a dryer.

An important relationship was also identified through the significant, positive relationship between present ownership of a dryer and intentions to purchase a dryer. This implies that farmers who presently own dryers have a higher probability of future purchases than those who do not presently own dryers.

This model also indicated that intentions to purchase drying capacity are more closely associated with those ecnomic variables whose values are easier and more likely to change in the near future; including farm size, percent of corn field shelled, and percent of corn sold at harvest. Farm type and tenancy are relatively stable over time and their coefficients were not significant in this model.

Implications of Results

Results of these two models indicate that farm dryers are most likely to be found on farms where the volume of field shelled corn is large enough to permit an economical operation, whose operator is relatively young, where the land tenure favors long-term investments, and where the shlled corn is to be stored on the farm for later use. In terms of geographical differences within Illinois, farm dryer sales may be expected to be greater in areas where livestock farms (rather than cash grain) are concentrated, or where increases in production and field shelling are relatively more rapid than the elevator's adjustments and expansion in drying and storage capacities.

It was found that farmers' purchases of drying and storage capacities are generally a joint decision. The advantages of on-farm drying are seldom realized without corresponding storage capacity. This impllies that the markets for these two products are complementary and sales should be oriented toward selling "grain handling systems" rather than separate drying and storage units.

Analysis of farmers' intentions to purchase implied that those farmrs who have reached their thresholds have already purchased dryers. Therefore, plans to buy drying equipment depend primarily on the anticipated changes in the explanatory variables. Among these explanatory variables, the most important possible changes are trends in field shelling and trends in farm size. Since the average percent of field shelling in Illinois has already reached 76 percent (1970) and in several areas of the state is approaching the maximum of 100 percent, the opportunities for expanding dryer sales through changes in this variable will diminish rapidly during the next few years.

Average farm size in Illinois has increased over time as indicated in Table 4. Much of this increase has been the result of relatively large farms adding smaller tracts to their curent holdings, rather than a merger of two small farms. The trend of increaed farm size will encourage adding capacity to existing drying facilities (or replacement of existing dryers with larger capacity equipment) rather than the purchase of dryers by new buyers in the market. Farms below the threshold size do not often become prospects for buying but their absorption by larger farms adds to the capacity requirement of the larger farms.

From the above observations and given that the percent of corn field shelled is approaching its upper limit, it is implied that the most important future market for farm dryers is likely to be in areas where farm consolidation requires expaned capacity or where obsolete equipment requires replacement. The manner in which farm size in Illinois increases as described in the preceeding paragraph may also explain why the results from the intention-model indicated that the anticipated increase in farm size is an important variable in future purchases, and that those farms presently owning dring equipment expressed a strong intention to purchase addtional (or replacement) drying equipment.

Summary

The statistical properties, the theoretical relationships, and the empirical illustration demonstrate the appropriateness of the multivariate probit model for problems where the dependent variable is dichotomous and where a threshold is evident in the decision process. The empirical example indicates the usefulness of the threshold concept and the probit technique in understanding economic decisions and behavior, in estimating market potential, and in identifying characteristics of market segments. Computer programs are now available to make this tool readily accessible for planning marketing strategies.

FIGURE 1

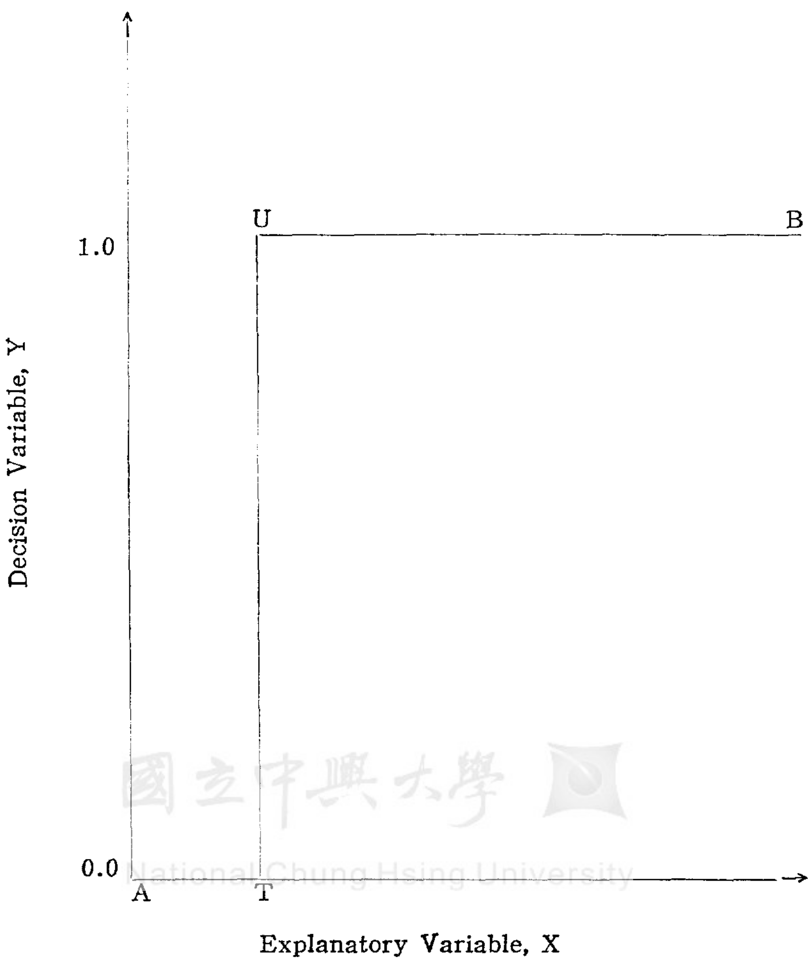

FIGURE 2

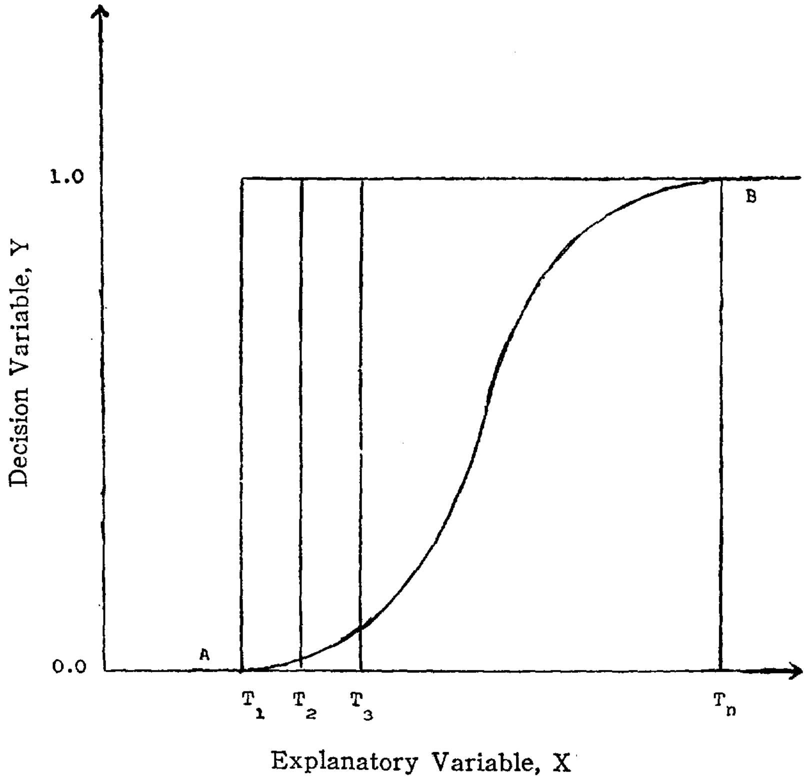
1.0
0.0
A
B
Decision Variable, Y
T1
T2
T3
Tn
Explanatory Variable, X

Table 1.

Description of the Variables in the Model

Variable	Definition	Measure	Unit
X_1	constant	$X_{11}=1$	1
X_2	size of enterprise	actual acres planted to corn in 1967	acres
X_3	type of farm	500 hogs or more=livestock=1, otherwise=cash grain=0	1 or 0
X_4	percent field shelled	$\frac{\text{acres field shelled in 1967}}{\text{acres harvested in 1967}} \times 100$	percent
X_5	degree of land ownership	if rent all=renter=1, if own some=owner-operator=0	1 or 0
X_6	Farm shelled corn storage capacity	actual on-farm shelled corn storage capacity	thousand bushels=1
X_7	percent sold at harvest	$\frac{\text{bu. sold from field in 1967}}{\text{bu. harvested in 1967}}$	percent
X_8	age	ages of farm operators divided into 6 classes	1—6
X_{MPO}	observed dryer ownership	own a farm dryer in 1967=1 not own a farm dryer in 1967=0	1 or 0
Y_{MPI}	intended dryer ownership	intend to purchase drying equipment in next 3 yre. from 1967, "yes"=1 "no" or "not sure"=0	1 or 0

Table 2.

Estimated Parameters From the Multivariate Probit Model

Explanatory Variables	Probit Maximum Likelihood Estimates ($\hat{\beta}_j$)	Standard Errors of $\hat{\beta}_j$	Calculated t-value	(−2) log λ(1)
X Constant	−1.47	0.28	−5.25	
X_2 Farm Size	0.0015	0.00061	2.46	
X_3 Farm Type	0.94	0.19	4.94	
X_4 Percent of Corn Field Saelled	0.0147	0.0015	9.80	
X_5 Tenancy	−0.219	0.131	−1.67	367.36
X_6 Farm Shelled Corn Storage Capacity	0.0439	0.0067	6.55	
X_7 Percent of Corn Sold at Harvest	−0.0066	0.0023	−2.88	
X_8 Age of Farmer	−0.113	0.054	−2.09	
Dependent variable $= Y_{MPO}$ (Table 1)				

(1) λ is the ratio of: $\frac{\log L\ (\hat{\beta}_1, 0, \cdots\cdots 0)}{\log L\ (\hat{\beta}_1, \hat{\beta}_2 \cdots\cdots \hat{\beta}_8)}$ (see text for detailed explanation).

Table 3.

Estimated Parameters from Future Intention Model

Explanatory Variables	Probit Maximum Likelihood Estimates ($\widehat{\beta}_j$)	Standard Errors of $\widehat{\beta}_j$	Calculated t-Value	(−2) log λ
X_1 Constant	−0.126	0.243	−0.52	
X_2 Farm Size	0.00109	0.00058	1.90	
X_3 Farm Type	0.184	0.178	1.03	
X_4 Percent or corn Field Shelled	0.0043	0.0016	2.69	
X_5 Tenancy	−0.215	0.125	1.72	88.84
X_6 Farm Shelled Corn Storage Capacity	−0.0062	0.0063	−0.98	
X_7 Percent of Corn Sold at Harvest	−0.0065	0.0024	−2.71	
X_8 Age of Farmer	−0.284	0.052	−5.46	
X_9 Dryer Ownership in 1967	0.325	0.135	2.40	
Dependent Varriable = Y_{MPI} (Table 1)				

Table 4.

Average Size of Farms in Illinois, 1950 to 1970

Year	Farms	Land in farms[10]	Av. size of farms
	Thous.	Thous. acres	Acres
1950	203	31,700	156
1951	198	31,600	160
1952	192	31,600	165
1953	186	31,500	169
1954	181	31,300	173
1955	178	31,300	176
1956	175	31,200	178
1957	172	31,100	181
1958	168	31,000	185
1959	164	30,900	188
1960	159	30,700	193
1961	155	30,600	198
1962	151	30,500	202
1963	148	30,400	206
1964	144	30,300	210
1965	141	30,200	216
1966	138	30,100	221
1967	132	30,000	226
1968	131	29,800	228
1969	128	29.700	232
1970	126	29,600	234

Source: "Illinois Agricultural Statistics," Annual Summary, 1970, Illinois Cooperative Crop Reporting Service, Illinois Department of Agriculture, U.S. Department of Agriculture, Bulletin 70—1.

FOOTNOTES

(1) Despite a similarity between the limited regressand problem and the threshold concept, a dependent variable that is inherently bounded by an upper and/or a lower limit does not necessarily imply imply the existence of a threshold.

(2) By the traditional assumption that $E(U_i)=0$.

(3) Since Y_t takes either 0 or 1, the values and the probabilities of U_t are:

Y_t	U_t	$f(U_t)$
0	$-X_t'\beta$	$1-X_t'\beta$
1	$1-X_t'\beta$	$X_t'\beta$

(4) $E(Y)\sum_{i=1}^{n} Y_t f(Y_t)=1\cdot f(Y_t=1)+0\cdot f(Y_t=0)$

$=1\cdot f(Y_t=1)+0$

$=f(Y_t=1)$

$=$the probabilities of those $Y_t=1$

(5) The estimation procedures of this model are mainly based on Finney's Probit Analysis and Tobin's extension.

(6) Refer to the next section for more detailed explanation.

(7) A farmer may sell corn directly to the elevator at a price discount for excess moisture or he may retain ownership of the corn while hiring the elevator to dry and store the grain. The net cost is approximately the same [11].

(8) The ecomic model and the rationale are more fully explained in [10, p. 10].

(9) The symbol A is equivalent to the term "Probits" in the traditional probit analysis [7]. If A^* is assumed to be $N(0,1)$ then the responsiveness of Y to changes in A decreases as the value of A moves either direction from 0 (in traditional probit analysis A^* is assumed to be $N(5, 1)$ and the responsive range centers on $A=5$). The threshold and the responsive range for A is thus observed to lie somewhere between -3 and $+3$ for $A^*\sim N(0,1)$ [between 2 and 8 for $A^*\sim N(5, 1)$]. The level of the threshold depends on the exact location of the curve for the particular set of data. The Probit-Probability Transformation Table is equivalent to the standard normal cumulative probability tables adjusted to a mean of 5 in stead of 0.

(10) Land in farms in 1910 U.S. Census was 32,523,000 acres.

REFERENCES

1. Berkson, J. "The Application of the Logistic Function to Bio-Assay." Journal of the American Statistical Association, Vol.39. pp. 357—65.
2. Berry, Brian J. L. ***Geography of Market Centers and Retail***

Distribution, Prentice-Hall, Inc., New Jersey, 1967.

3. Bilkey, Warren J., *The Basic Relationship in Consumer Expenditure Behavior*, Harvard Studies in Marketing Farm Products No. 4—H, Cambridge, 1951, pp. 66.
4. Brennan, J.F., "Evaluation of Parameters in the Gompertz and Makeham Equations," Journal of American Statistical Association, Vol. 44, pp. 116—21.
5. Dagenais, Marcel G., "A Threshold Regression Model," *Econometrica*, Vol. 37, April 1969, pp. 193—203.
6. Farrell, M.J., "The Demand for Motor Cars in the United States," *Journal of the Royal Statistical Society*, Vol. 117, Part 2, 1954, pp. 171—201.
7. Finney, D. J., *Probit Analysis*, Cambridge University Press, Cambridge, England, 1952.
8. Goldberger, Arthur S., *Econometric Theory*, John Wiley and Sons, Inc., New York 1964.
9. Heckel, R.V. and Jordan, R.M., *Psychology*, The C.V. Mosby Company, St. Louis, 1967.
10. Hill. L.D., *Adjustments in Drying and Storage Capacity at Farm and Elevator*, University of Illinois, Department of Agriculture Economics, AERR 105, July 1970.
11. Hill, L. D., "Buying Corn on a No.2 Basis: A Solution to the Moisture-Discount Problem," *Economics for Agriculture*, Mktg. 49, *Univ. of Ill.*
12. Isaacson, R.L.; Hutt, M.L.; Blum M.L., *Psychology: The Science of Behavior*, Harper and Row Publishers, New York, 1965.
13. Johnson, G. L.; Halter, A. N.; Jensen, H. R.; Thomas, D. W., *A Study of Managerial Processes of Midwestern Farmers*, The Iowa State University Press, Ames, Iowa, 1961.
14. Johnston, J., *Econometric Methods*, McGraw-Hill Book Company Inc., New York, 1963.
15. Knight, Frank H., *Risk Uncertainty* and Profit, Harper and Row Publishers, New York, 1965.
16. Mood, A. M., *Introduction to the Theory of Statistics*, McGraw-Hill, New York, 1950.
17. Newman, J. W., and Staelin, R., "Multivariate Analysis of Differences in Buyer Decision Time," *Journal of Marketing*

Research, May. 1971, pp. 192—198.

18. Nourse, H.O., ***Regional Economics***, McGraw-Hill Book Company, New York 1968.
19. Rosett, R. N., "A statistical Model of Friction in Economics," ***Econometrica***, Vol. 27, No.2, April, 1959, pp. 263—267.
20. Simon, H. A., "Theories of Decision-Making Economics and Behavioral Sciences." ***American Economic Review***, Vol. XLIX, No. 3, June 1959, pp. 253—281.
21. Tobin, James, "The Application of Multivariate Probit Analysis to Economic Survey Data," Cowles Fundation Discussion Paper No. 1, 1955.
22. Tobin, James, "Estimation of Relationships for Limited Dependent Variable," ***Econmetrica***, Vol. 26, January, 1958, pp. 24—36.
23. Tomek, W. G., "Regression Analysis with a Limited Dependent Variable," ***Journal of Farm Economics*** Vol. 50, May 1968, pp. 445—447.
24. Warner, S. L., ***Stochastic Choice of Mode in Urban Travel: A Study in Binary Choice***, Northwestern University Press, 1962.
25. Zellner, A., and Lee, Tong-Hun, "Joint Estimation of Relationships Involving Discrete Random Variables," ***Econometrica***, Vol. 33, April, 1965. pp. 382—394.